

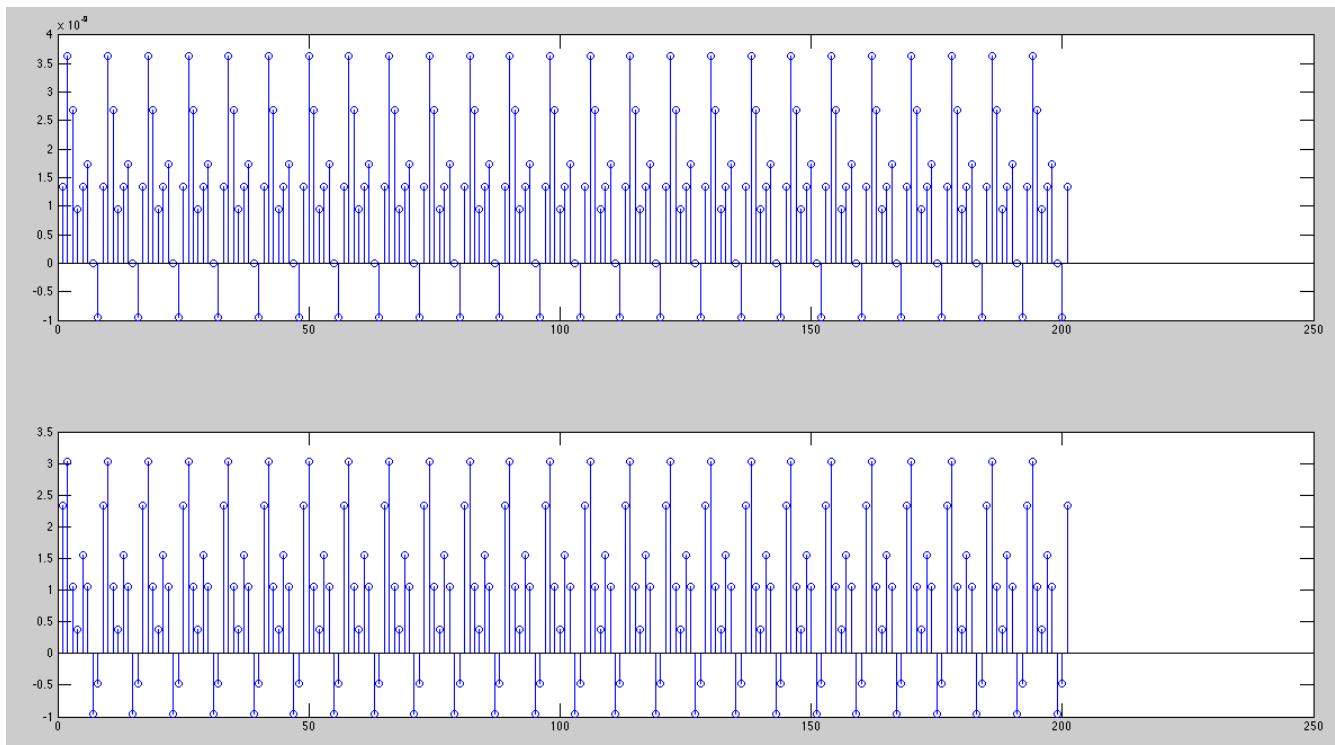
Problem 5.45 (c) in Fundamentals of Signals and Systems asks for the response, $y[n]$, of a discrete time system given the input $x[n] = 1 + \sin((\pi/4)n) + \sin((\pi/2)n)$ and the impulse response $h[n] = 1.9(-0.9)^n u[n]$. I first noted that the response of a system can be computed by taking the convolution of the input, $x[n]$, and the impulse response, $h[n]$. I then found equation 5.65 in section 5.5.1 on page 250 in the text and decided that this was an appropriate method of solution for this problem.

$$y[n] = A|H(\Omega_0)|\cos(\Omega_0 n + \theta + \angle H(\Omega_0)), \quad n = 0, \pm 1, \pm 2, \dots \quad (5.65)$$

I computed $H(\Omega)$ by taking the Discrete Time Fourier Transform of $h[n]$. Then, I considered that $y[n]$ is equal to the sum of responses to $x_1[n] = 1$, $x_2[n] = \sin((\pi/4)n)$, and $x_3[n] = \sin((\pi/2)n)$. Next, I found the magnitude and phase angle of $H(\Omega_0)$ for $\Omega_0 = 0$, $\pi/4$, and $\pi/2$. Using equation 5.65 and considering that A is 1 for $x_1[n]$, $x_2[n]$, and $x_3[n]$ I proceeded to compute the response to the system, $y[n]$. Finally, I arrived at the following solution:

$$y[n] = 1 + 1.08\sin((\pi/4)n + 0.371) + 1.41\sin((\pi/2)n + 0.733)$$

I verified this solution by writing a MATLAB script that computes the response, $y[n]$, by taking the convolution ($\text{conv}(x[n], h[n])$) of $x[n]$ and $h[n]$, plots it (using $\text{stem}(y[n])$), and then plots the response that I obtained using equation 5.65 for comparison. The MATLAB script arbitrarily generates a vector of integers to represent n from 0 to 200. It then implements $x[n]$ using this n vector and implements $h[n]$ using a for loop. After that it computes the response, $y[n]$, using the conv function to take the convolution of $x[n]$ and $h[n]$. Finally, it plots the response computed in the script and then the response that I determined analytically using the subplot and stem functions. The plots of the two system responses is shown below.



Stem plots of the MATLAB response (top) and analytical response (bottom)