

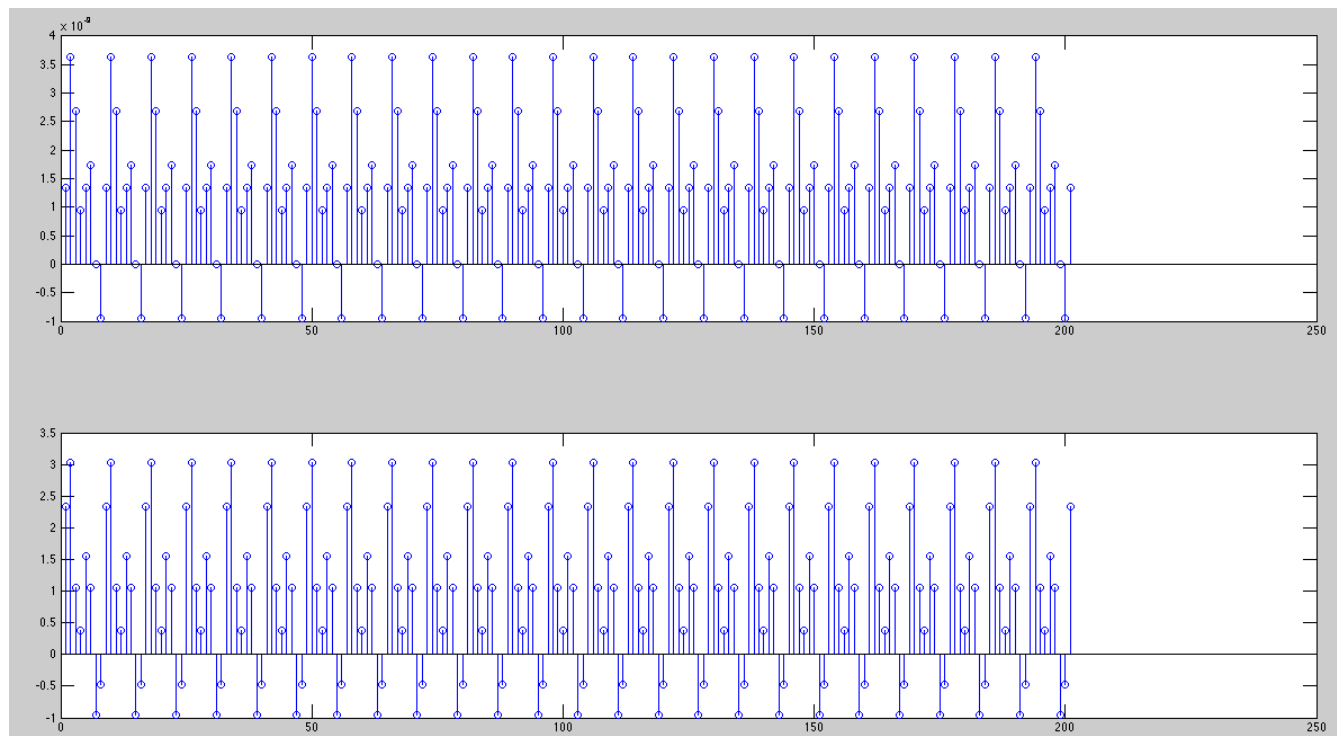
Problem 5.45 (c) in Fundamentals of Signals and Systems asks for the response,  $y[n]$ , of a discrete time system given the input  $x[n] = 1 + \sin((\pi/4)n) + \sin((\pi/2)n)$  and the impulse response  $h[n] = 1.9(-0.9)^n u[n]$ . I first noted that the response of a system can be computed by taking the convolution of the input,  $x[n]$ , and the impulse response,  $h[n]$ . I then found equation 5.65 in section 5.5.1 on page 250 in the text and decided that this was an appropriate method of solution for this problem.

$$y[n] = A|H(\Omega_0)|\cos(\Omega_0 n + \theta + \angle H(\Omega_0)), \quad n = 0, \pm 1, \pm 2, \dots \quad (5.65)$$

I computed  $H(\Omega)$  by taking the Discrete Time Fourier Transform of  $h[n]$ . Then, I considered that  $y[n]$  is equal to the sum of responses to  $x_1[n] = 1$ ,  $x_2[n] = \sin((\pi/4)n)$ , and  $x_3[n] = \sin((\pi/2)n)$ . Next, I found the magnitude and phase angle of  $H(\Omega_0)$  for  $\Omega_0 = 0, \pi/4$ , and  $\pi/2$ . Using equation 5.65 and considering that  $A$  is 1 for  $x_1[n]$ ,  $x_2[n]$ , and  $x_3[n]$  I proceeded to compute the response to the system,  $y[n]$ . Finally, I arrived at the following solution:

$$y[n] = 1 + 1.08\sin((\pi/4)n + 0.371) + 1.41\sin((\pi/2)n + 0.733)$$

I verified this solution by writing a MATLAB script that computes the response,  $y[n]$ , by taking the convolution ( $\text{conv}(x[n], h[n])$ ) of  $x[n]$  and  $h[n]$ , plots it (using  $\text{stem}(y[n])$ ), and then plots the response that I obtained using equation 5.65 for comparison. The MATLAB script arbitrarily generates a vector of integers to represent  $n$  from 0 to 200. It then implements  $x[n]$  using this  $n$  vector and implements  $h[n]$  using a for loop. After that It computes the response,  $y[n]$ , using the  $\text{conv}$  function to take the convolution of  $x[n]$  and  $h[n]$ . Finally, it plots the response computed in the script and then the response that I determined analytically using the subplot and  $\text{stem}$  functions. The plots of the two system responses is shown below.



*Stem plots of the MATLAB response (top) and analytical response (bottom)*