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Extra Credit
Test Two
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Part One: Theoretical Solution

$x[n] = 1 + \sin\left(\frac{\pi n}{4}\right) + \sin\left(\frac{\pi n}{2}\right)$ $\qquad \cancel{h[n] = 1.9(-0.9)^n u[n]}$

$h[n] = 1.9(-0.9)^n u[n] \longleftrightarrow H(\omega) = \frac{1.9 e^{j\Omega}}{e^{j\Omega} + 2.9}$ DTFT

Break it up $\cancel{x_1[n] = 1}$

$x_1[n] + x_2[n] + x_3[n]$

$x_1[n] = 1$

$$X_1(\omega) = \sum_{k=\infty}^{\infty} 2\pi \, \delta(\omega - 2\pi k)$$

$$Y_1(\omega) = X(\omega) H(\omega)$$

$$= \sum_{k=\infty}^{\infty} 2\pi \, H(\omega) \, \delta(\omega - 2\pi k) \qquad C = -2\pi k$$

$$-C = 2\pi k$$

$$C = 0$$

$$= \sum_{k=\infty}^{\infty} 2\pi \, \overset{1}{H(0)} \, \delta(\omega - 2\pi k)$$

$$= \sum_{k=-\infty}^{\infty} 2\pi \, \delta(\omega - 2\pi k)$$

$y_1[n] = 1$

Figure 1

$X_2[n] = \sin\left(\frac{\pi n}{4}\right) \qquad w_0 = \frac{\pi}{4}$

$X_2(\omega) = \sum_{K=-\infty}^{\infty} j\pi\left[\delta\left(\omega+\frac{\pi}{4}-2\pi K\right) - \delta\left(\omega-\frac{\pi}{4}-2\pi k\right)\right]$

$\theta = \angle H\left(\frac{\pi}{4}\right)$

$w_0 = \frac{\pi}{4}$

$Y_2(\omega) = X_2(\omega) H(\omega)$

$= \sum_{K=-\infty}^{\infty} j\pi\left[H(\omega)\,\delta\left(\omega+\frac{\pi}{4}-2\pi K\right) - H(\omega)\,\delta\left(\omega-\frac{\pi}{4}-2\pi K\right)\right]$

$= \sum_{K=-\infty}^{\infty} j\pi\left[H\left(\frac{-\pi}{4}\right)\delta\left(\omega+\frac{\pi}{4}-2\pi k\right) - H\left(\frac{\pi}{4}\right)\delta\left(\omega-\frac{\pi}{4}-2\pi K\right)\right]$

$= \sum_{K=-\infty}^{\infty} j\pi\left[\left|H\left(\frac{-\pi}{4}\right)\right| e^{-j\angle H\left(\frac{\pi}{4}\right)}\delta\left(\omega+\frac{\pi}{4}-2\pi k\right) - \left|H\left(\frac{\pi}{4}\right)\right| e^{j\angle H\left(\frac{\pi}{4}\right)}\delta\left(\omega-\frac{\pi}{4}-2\pi\right.\right.$

$Y_2[n] = \left|H\left(\frac{\pi}{4}\right)\right| \sin\left(\frac{\pi}{4}n + \angle H\left(\frac{\pi}{4}\right)\right)$

$1.08214 \sin\left(\frac{\pi}{4}n + 0.370902\right)$

$X_3[n] = \sin\left(\frac{\pi n}{2}\right) \qquad w_0 = \frac{\pi}{2}$

$X_3(\omega) = \sum_{K=-\infty}^{\infty} j\pi\left[\delta\left(\omega+\frac{\pi}{2}-2K\pi\right) - \delta\left(\omega-\frac{\pi}{2}-2K\pi\right)\right]$

$\theta = \angle H\left(\frac{\pi}{2}\right)$

$w_0 = \frac{\pi}{2}$

$Y_3(\omega) = X_3(\omega) H(\omega)$

$= \sum_{K=-\infty}^{\infty} j\pi\left[H(\omega)\,\delta\left(\omega+\frac{\pi}{2}-2K\pi\right) - H(\omega)\,\delta\left(\omega-\frac{\pi}{2}-2K\pi\right)\right]$

$= \sum_{K=-\infty}^{\infty} j\pi\left[H\left(\frac{\pi}{2}\right)\delta\left(\omega+\frac{\pi}{2}-2K\pi\right) - H\left(\frac{\pi}{2}\right)\delta\left(\omega-\frac{\pi}{2}-2K\pi\right)\right]$

$= \sum_{K=-\infty}^{\infty} j\pi\left[\left|H\left(\frac{\pi}{2}\right)\right| e^{-j\angle H\left(\frac{\pi}{2}\right)}\delta\left(\omega+\frac{\pi}{2}-2K\pi\right) - \left|H\left(\frac{\pi}{2}\right)\right| e^{j\angle H\left(\frac{\pi}{2}\right)}\delta\left(\omega-\frac{\pi}{2}-2K\right.\right.$

$Y_3[n] = \left|H\left(\frac{\pi}{2}\right)\right| \sin\left(\frac{\pi}{2}n + \angle H\left(\frac{\pi}{2}\right)\right)$

$Y_3[n] = 1.41421 \sin\left(\frac{\pi}{2}n + .732815\right)$

$Y[n] = 1 + 1.08214 \sin\left(\frac{\pi}{4}n + 0.370902\right) + 1.41421 \sin\left(\frac{\pi}{2}n + .73281\right.$

Figure 2

(4)

1) $\cos^{1}\theta + i\sin^{-1}\theta = e^{i\theta}$

2) $H(\omega) = 1 + e^{-j\omega}$

$H(\Omega)\delta(\Omega+c) = H(-c)\,\delta(\Omega+c)$

$|H(-\Omega)| = |H(\Omega)|$

3) $H(-\Omega_0) = |H(\Omega_0)|\, e^{-j\angle H(\Omega_0)}$

$H(\Omega_0) = |H(\Omega_0)|\, e^{j\angle H(\Omega_0)}$

Figure 3

Part Two: Verification Solution using MATLAB

```
EDU>> n=0:0.5:150;
EDU>> x=1+sin((pi*n)/4)+sin((pi*n)/2);
EDU>>  for k=1:15;
h(k)=1.9*(-0.9)^(k-1);
end;
EDU>> z=conv(x,h);
EDU>> stem(z)
EDU>> y=1+(1.08214*sin(((pi*n)/4)+0.370902))+(1.41421*sin(((pi*n)/2)+0.732815));
EDU>> stem(y)
EDU>> hold
Current plot held
EDU>> stem(z)
```

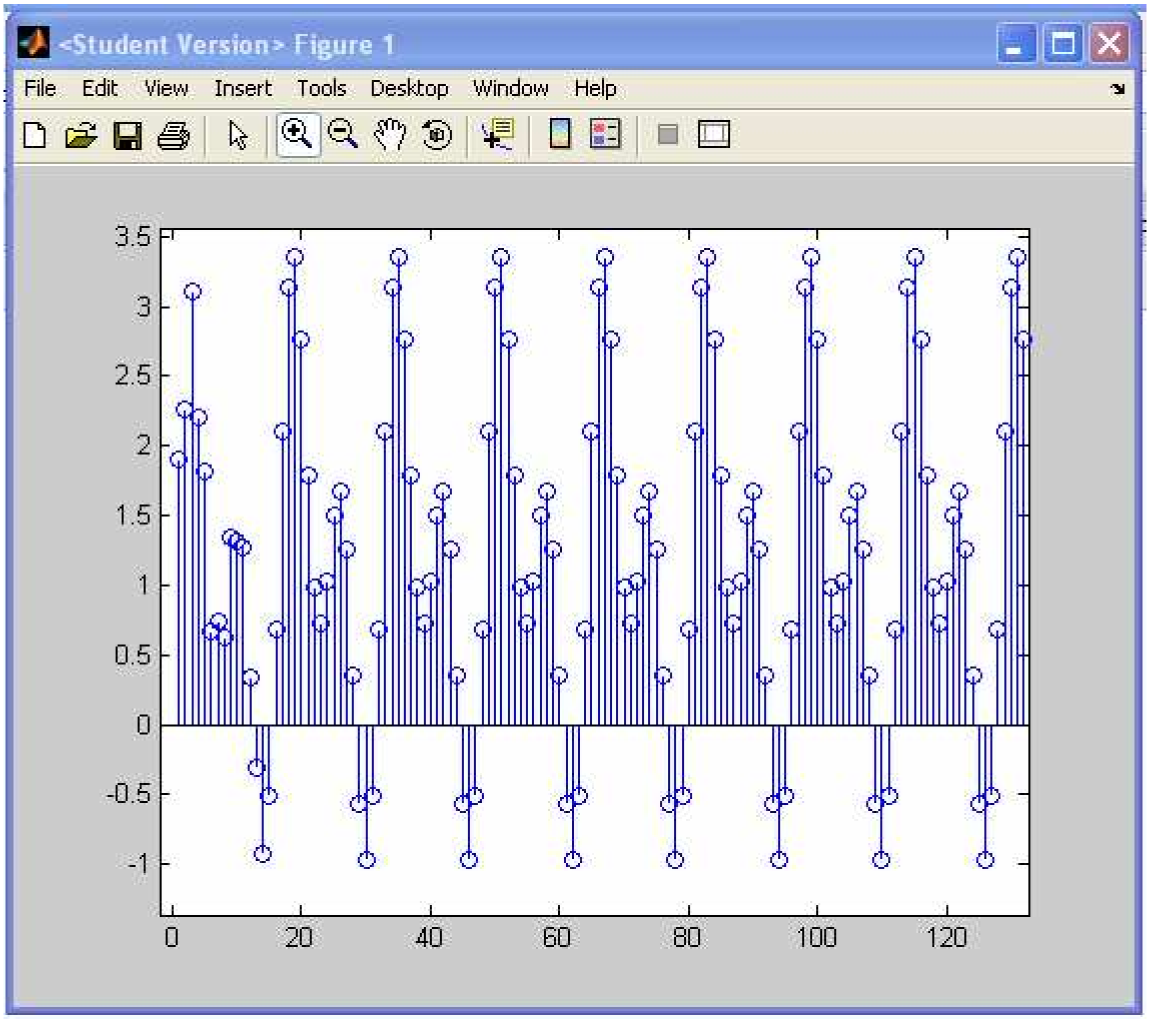

Figure 4 stem(z)

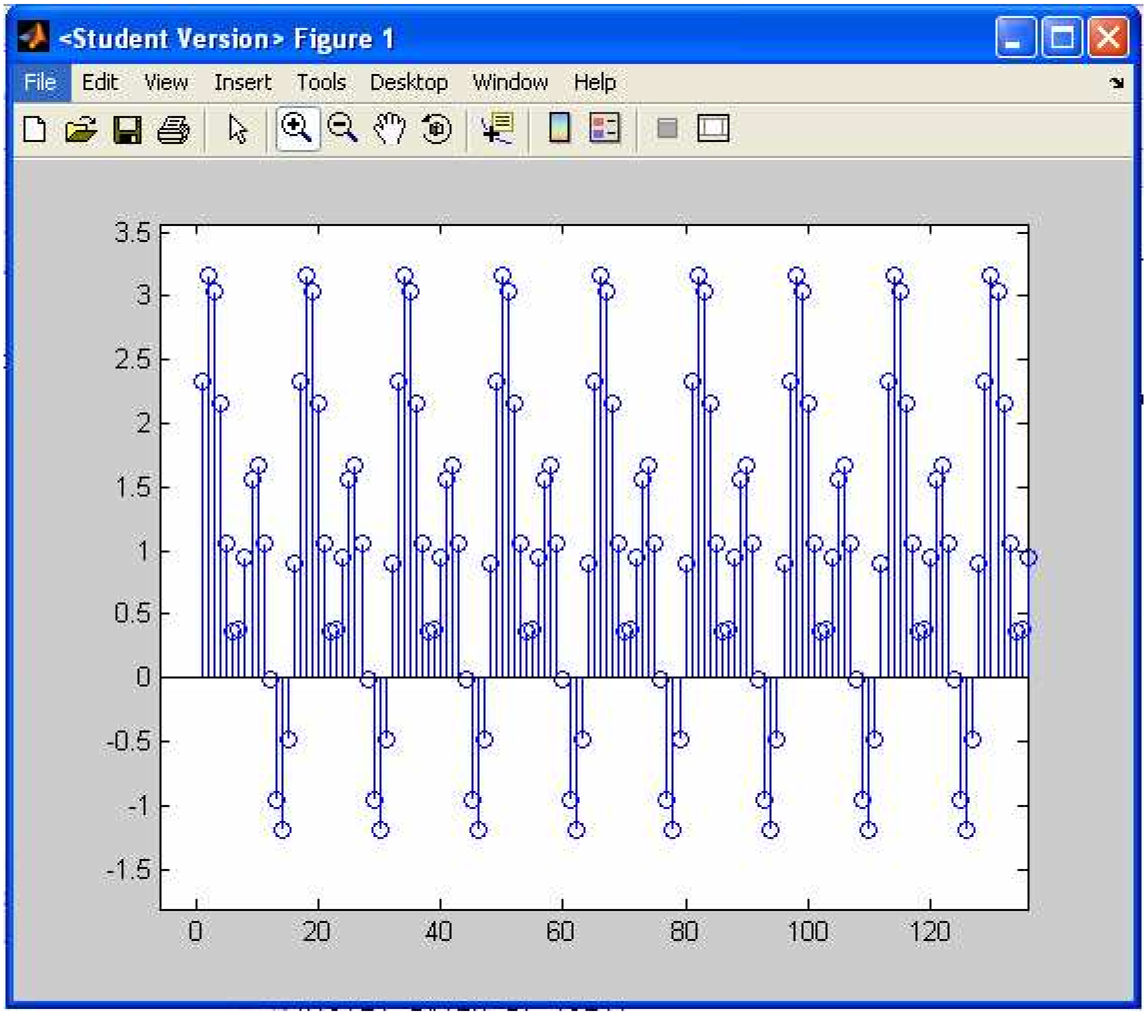

Figure 5 stem(y)

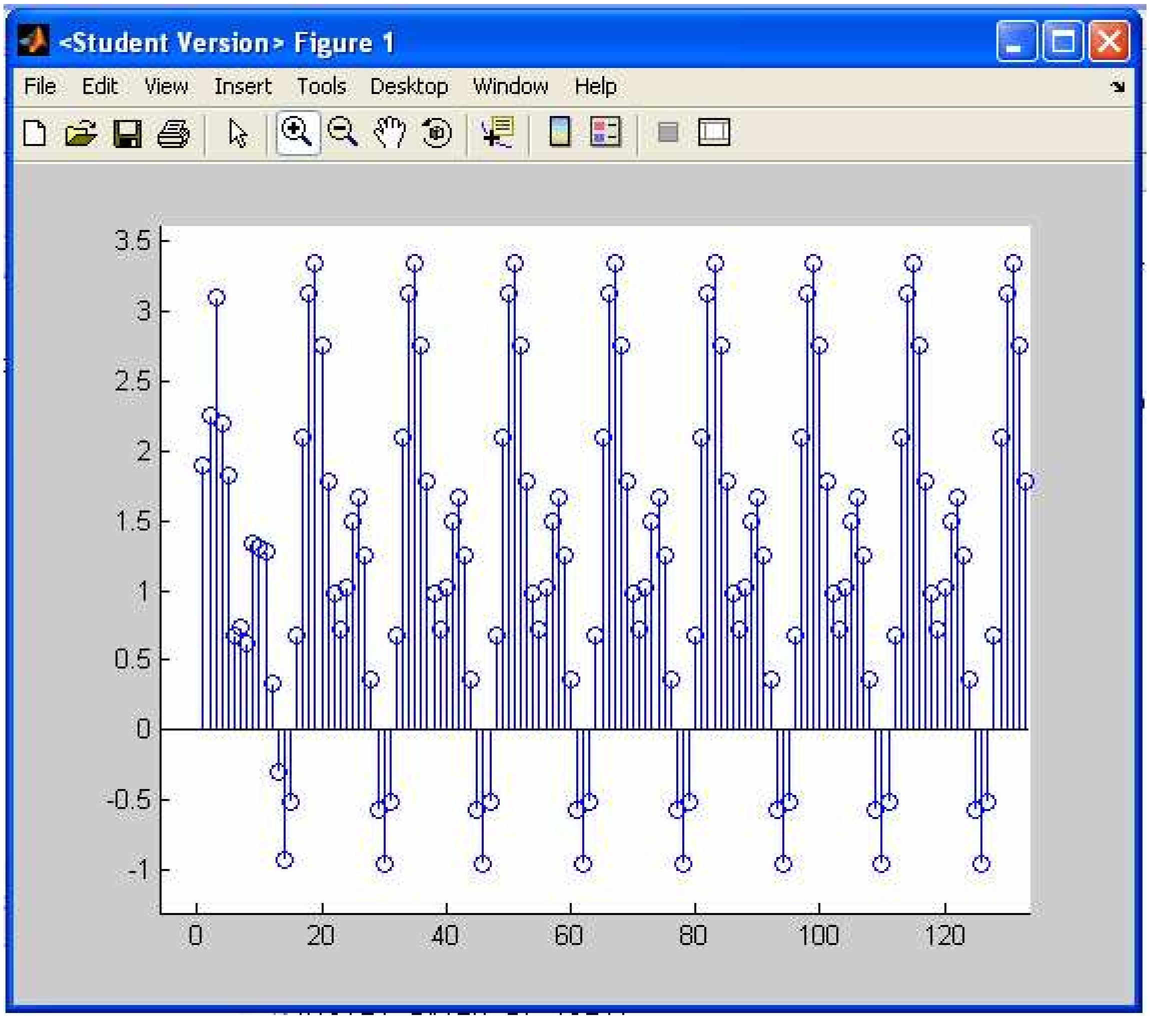

Figure 6 stem(z)

Part Three: Description of Solution.

The process for solving the problem 5.45c is as follows: compute analytically, then take the resulting analytical equation and compare it to the results of allowing Matlab to solve the same problem. Figure 4 and 6 are the resulting graphs. Figure 6 shows the analytical result and 4 shows the matlab computation.

Problem 5.45c would have been very cumbersome to work in the time domain so it was instead worked in the frequency domain. After this decision was made, things became more straight forward. Figure 3 shows the three equations from chapter 5 that were used. In conjunction with these equations, the DTFT tables were also used.

This method best resembles the last method described by the instructor. The input signal is the combination of a DC signal and two sinewaves, so the DC value can be computed using the difference equation (given) and the output of the sinewaves can be computed by evaluating the system's frequency response.