→ $x[n] = 1 + \sin(\pi n/2) + \sin(\pi n/4)$; $h[n] = 1.9(-0.9)^n u[n]$

→ Represent $x[n]$ as a sum of sinusoids: $x_1[n] = 1 = \cos(0n)$

$x_2[n] = \sin(\pi n/2)$

$x_3[n] = \sin(\pi n/4)$

→ Take the DTFT of $x_k[n]$ and $h[n]$: $X_1(\Omega) = \sum_{k=-\infty}^{\infty} \pi[\delta(\Omega - 2\pi k) + \delta(\Omega - 2\pi k)]$

$$X_2(\Omega) = \sum_{k=-\infty}^{\infty} j\pi[\delta(\Omega + \pi/2 - 2\pi k) - \delta(\Omega - \pi/2 - 2\pi k)]$$

$$X_3(\Omega) = \sum_{k=-\infty}^{\infty} j\pi[\delta(\Omega + \pi/4 - 2\pi k) - \delta(\Omega - \pi/4 - 2\pi k)]$$

$$H(\Omega) = \frac{1.9}{1 + 0.9e^{-j\Omega}}$$

→ To find $Y(\Omega)$, multiply $X(\Omega)$ and $H(\Omega)$:

$$Y_1(\Omega) = X_1(\Omega)H(\Omega) = \sum_{k=-\infty}^{\infty} \pi H(\Omega)[\delta(\Omega - 2\pi k) + \delta(\Omega - 2\pi k)]$$

$$Y_2(\Omega) = X_2(\Omega)H(\Omega) = \sum_{k=-\infty}^{\infty} j\pi H(\Omega)[\delta(\Omega + \pi/2 - 2\pi k) - \delta(\Omega - \pi/2 - 2\pi k)]$$

$$Y_3(\Omega) = X_3(\Omega)H(\Omega) = \sum_{k=-\infty}^{\infty} j\pi H(\Omega)[\delta(\Omega + \pi/4 - 2\pi k) - \delta(\Omega - \pi/4 - 2\pi k)]$$

→ since $H(\Omega)\delta(\Omega + c) = H(-c)\delta(\Omega + c)$, $Y(\Omega)$ can be rewritten as:

$$Y_1(\Omega) = \sum_{k=-\infty}^{\infty} \pi[H(2\pi k)\delta(\Omega - 2\pi k) + H(2\pi k)\delta(\Omega - 2\pi k)]$$

$$Y_2(\Omega) = \sum_{k=-\infty}^{\infty} j\pi\left[H(-\pi/2 + 2\pi k)\delta(\Omega + \pi/2 - 2\pi k) - H(\pi/2 + 2\pi k)\delta(\Omega - \pi/2 - 2\pi k)\right]$$

$$Y_3(\Omega) = \sum_{k=-\infty}^{\infty} j\pi[H(-\pi/4 + 2\pi k)\delta(\Omega + \pi/2 - 2\pi k) - H(\pi/4 + 2\pi k)\delta(\Omega - \pi/4 - 2\pi k)]$$

→ Since $H(\Omega)$ is periodic and $T=2\pi$, $H(-\Omega_0+2\pi k)=H(-\Omega_0)$ and $H(\Omega_0+2\pi k)=H(\Omega_0)$,

also, since the transfer function is real, $|H(-\Omega_0)|=|H(\Omega_0)|$

$$\angle H(-\Omega) = -\angle H(\Omega)$$

so by converting the outputs $Y(\Omega)$ to polar form:

$$Y_1(\Omega)=\sum_{k=-\infty}^{\infty}\pi|H(0)|\left[e^{-j\angle H(0)}\delta(\Omega-2\pi k)+e^{j\angle H(0)}\delta(\Omega-2\pi k)\right]$$

$$Y_2(\Omega)=\sum_{k=-\infty}^{\infty}j\pi|H(\pi/2)|\left[e^{-j\angle H(\pi/2)}\delta(\Omega+\pi/2-2\pi k)-e^{j\angle H(\pi/2)}\delta(\Omega-\pi/2-2\pi k)\right]$$

$$Y_3(\Omega)=\sum_{k=-\infty}^{\infty}j\pi|H(\pi/4)|\left[e^{-j\angle H(\pi/4)}\delta(\Omega+\pi/4-2\pi k)-e^{j\angle H(\pi/4)}\delta(\Omega-\pi/4-2\pi k)\right]$$

→ Find the amplitudes and phase shifts of $H(\Omega)$:

$$|H(\Omega)|=\frac{1.9}{\sqrt{(1+0.9\cos\Omega)^2+0.9^2\sin^2(\Omega)}}=\frac{1.9}{\sqrt{1.81+1.8\cos\Omega}}$$

$$\angle H(0)=\frac{1.9}{1.9}=0 \text{ rad}$$

$$\Rightarrow |H(0)|=\frac{1.9}{\sqrt{3.61}}=1$$

$$\angle H(\pi/2)=\frac{1.9j}{0.9+j}=0.733 \text{ rad}$$

$$|H(\pi/2)|=\frac{1.9}{\sqrt{1.81}}=\sqrt{2}=1.41$$

$$\angle H(\pi/4)=\frac{\frac{1.9\sqrt{2}}{2}+j\frac{1.9\sqrt{2}}{2}}{(\frac{\sqrt{2}}{2}+0.9)+\frac{\sqrt{2}}{2}j}=0.371 \text{ rad}$$

$$|H(\pi/4)|=\frac{1.9}{\sqrt{1.81+0.9(\sqrt{2})}}=1.08$$

→ substitute $|H(\Omega)|$ and $\angle H(\Omega)$ values and take the inverse DTFT of $Y(\Omega)$

$$Y_1(\Omega)=\sum_{k=-\infty}^{\infty}\pi\left[\delta(\Omega-2\pi k)+\delta(\Omega-2\pi k)\right]$$

$$Y_2(\Omega)=\sum_{k=-\infty}^{\infty}j\pi\sqrt{2}\left[e^{-0.733j}\delta(\Omega+\pi/2-2\pi k)-e^{0.733j}\delta(\Omega-\pi/2-2\pi k)\right]$$

$$Y_3(\Omega)=\sum_{k=-\infty}^{\infty}j\pi\, 1.08\left[e^{-0.371j}\delta(\Omega+\pi/4-2\pi k)-e^{0.371j}\delta(\Omega-\pi/4-2\pi k)\right]$$

$y_1[n] = \cos(0n) = 1$

$y_2[n] = 1.41 \sin(\pi n/2 + 0.733)$

$y_3[n] = 1.08 \sin(\pi n/4 + 0.371)$

→ The output response is the sum of the outputs due to inputs $x_1[n]$, $x_2[n]$, and $x_3[n]$:

$\therefore\ y[n] = 1 + 1.41 \sin(\pi n/2 + 0.733) + 1.08 \sin(\pi n/4 + 0.371)$