

March 26, 2009

$$y[n+1] + .9 y[n] = 1.9 x[n+1]$$

$$= h[n+1] + .9 h[n] = 1.9 \delta[n+1]$$

given $x[n] = 1 + \sin(\frac{\pi}{4}n) + \sin(\frac{\pi}{2}n)$

$$= 1.9 (-.9)^{n+1} u[n+1] + .9 (1.9) (-.9)^n u[n]$$

$$= 1.9 (-.9)^{n+1} (u[n+1] - u[n])$$

$$h[n] = 1.9 (-.9)^n u[n]$$

$n = -1$ if not it is equal to 0
 so, $(-.9)^{n+1} = 1$
 so, $u[n+1] - u[n] = \delta[n+1]$

$$= 1.9 (-.9)^{n+1} (u[n+1] - u[n])$$

$$\delta[n+1]$$

so,

$$= 1.9 (\delta[n+1])$$

Previously found

$$H(\omega) = \frac{-1.9 e^{j\omega}}{e^{j\omega} + .9}$$

so,

$$X(\omega) = 2\pi \delta(\omega) + \delta\pi [\delta(\omega + \frac{\pi}{4}) - \delta(\omega - \frac{\pi}{4})] + \gamma\pi [\delta(\omega + \frac{\pi}{2}) - \delta(\omega - \frac{\pi}{2})]$$

$$Y(\omega) = H(\omega) X(\omega)$$

$$Y[n] = |H(\omega)| \sin(\frac{\pi}{4}n + \angle H(\frac{\pi}{4})) + |H(\frac{\pi}{2})| \sin(\frac{\pi}{2}n + \angle H(\frac{\pi}{2}))$$

so then, Find $H(0), H(\frac{\pi}{4}), H(\frac{\pi}{2})$

$$H(0) = \frac{1.9}{1+.9} = 1 \quad H(\frac{\pi}{4}) = \frac{1.9 e^{j\pi/4}}{e^{j\pi/4} + .9} \quad H(\frac{\pi}{2}) = \frac{1.9 e^{j\pi/2}}{e^{j\pi/2} + .9}$$

$$H(\frac{\pi}{4}) = \frac{1.9 (\cos(\frac{\pi}{4}) + j \sin(\frac{\pi}{4}))}{\cos(\frac{\pi}{4}) + j \sin(\frac{\pi}{4}) + .9} = \frac{1.9 (.707 + j.707)}{1.607 + j.707} = \frac{1.34 + j.134}{1.6 + j.707} = \frac{1.08 = |H(\omega)|}{.37 = \angle H(\frac{\pi}{4})}$$

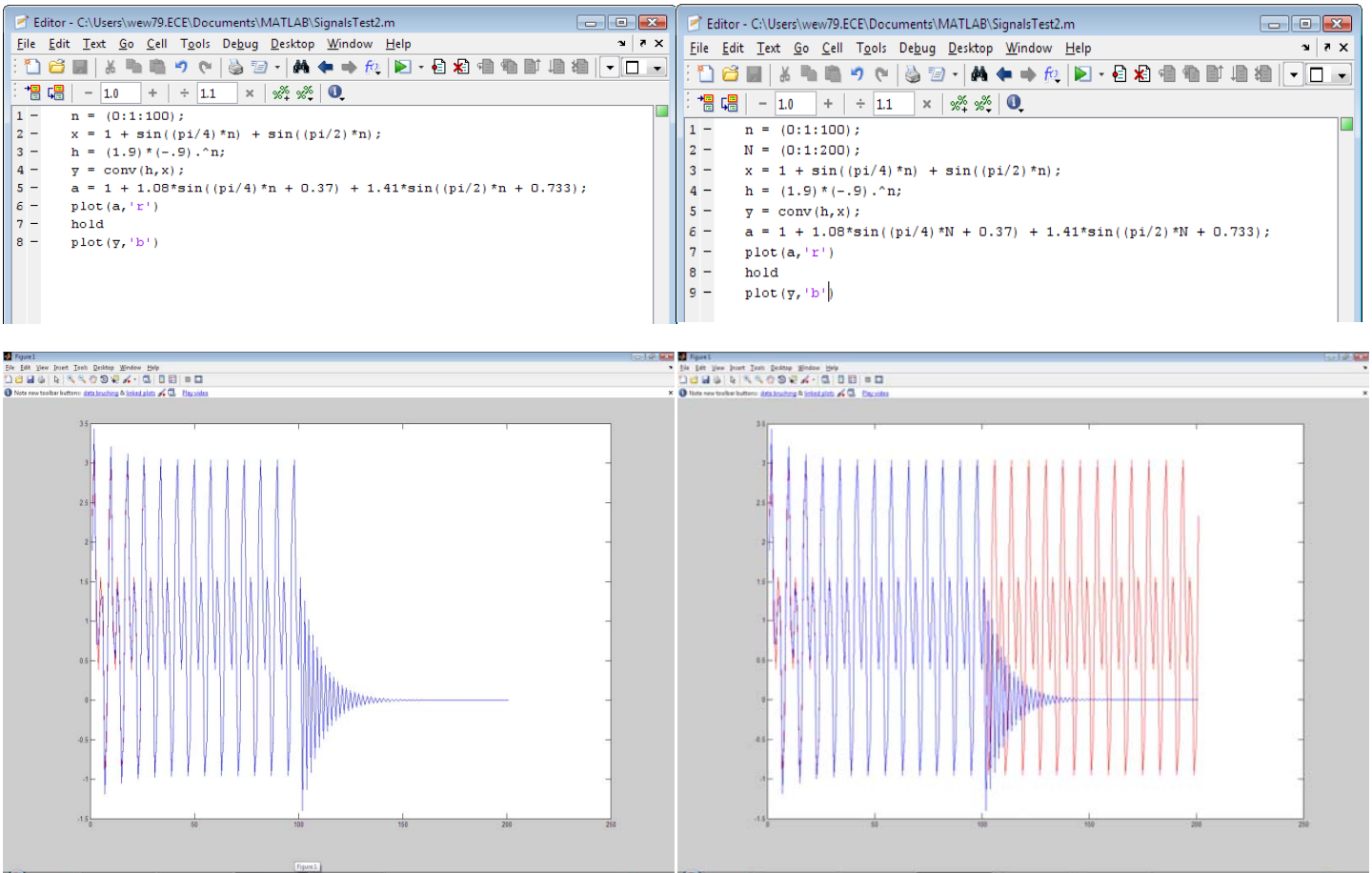
$$H(\frac{\pi}{2}) = \frac{1.9 (\cos(\frac{\pi}{2}) + j \sin(\frac{\pi}{2}))}{\cos(\frac{\pi}{2}) + j \sin(\frac{\pi}{2}) + .9} = \frac{1.9 (0 + j)}{.9 + j} = \frac{1.9 j}{.9 + j} = \frac{1.41 = |H(\omega)|}{.733 = \angle H(\frac{\pi}{2})}$$

$$Y[n] = 1 + 1.08 \sin(\frac{\pi}{4}n + .37) + 1.41 \sin(\frac{\pi}{2}n + .733)$$

$$Y[n] = 1 + 1.08 \sin(\frac{\pi}{4}n + .37 \text{ rad}) + 1.41 \sin(\frac{\pi}{2}n + .733 \text{ rad})$$

$\sin = \text{Sih}$

Matlab



At first I tried to do a for loop with an array so it would be really in discrete values, but it didn't work out so well. Notice that analytical solution is plotted in red and is represented by "a". The matlab convolution is represented by "y" and is plotted in blue. As you can see both answers overlap after at around $n=40$ until $n=100$. So, I believe that they both represent the same answer. The only thing that bothers me is the fact that the convolution adds the $h[n]$ and $x[n]$ making the plot $n=200$. So in the right Editor screen shot I made another array that was $N=200$. The plot below it shows the new adjustment. This bothers me greatly mostly because I don't really understand what the right answer should look like but I have a much greater understanding of the convolution of $h[n]$ and $x[n]$. Hopefully if my solution isn't correct you can see that I do understand the material, and I think that should be the most important. Hopefully you will too.

Written Explanation

One way to get a linear time-invariant discrete-time system you must have an $h[n]$, unit-pulse response, and an $x[n]$, input signal. By equations 5.58 and 5.59 on page 249 in our Fundamentals of Signals and Systems book, you can see that the convolution of $h[n]$ and $x[n]$ equal $y[n]$, the output response. Once there you can use the DTFTs of the input response and output response convolution to represent the output. This can also be represented as the product of the DTFTs of $x[w]$ and $h[w]$. The output response has two components, the first is the magnitude, and the second is the angle. The magnitude is found from the magnitudes of $x[w]$ and $h[w]$. The angle is found by the angle of $x[w]$ and $h[w]$. Once you have your none-specific $y[n]$ you need to find your values for $h[w]$ which in my case were $h(0)$, $h(\pi/4)$, $h(\pi/2)$. From there you can plug in your magnitude and angle to the $y[n]$ you got from your convolution. One other thing to watch for is that your angle will first be in degrees and needs to be converted to radians. That should be it.