Brandon Hoda

Test two extra credit
Signals and Systems
Dr Joseph Picone
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Analytical Solution:

5.45 $y[n+1] + 0.9y[n] = 1.9x[n+1]$

$h[n] = 1.9(-.9)^n u[n]$ so $H(\omega) = \frac{1.9}{1+.9e^{-j\omega}} \rightarrow \frac{1.9e^{j\omega}}{e^{j\omega}+0.9}$

$X(\omega) = 2\pi\delta(\omega) + j\pi[\delta(\omega + \frac{\pi}{4}) - \delta(\omega - \frac{\pi}{4})] +$

$j\pi[\delta(\omega + \frac{\pi}{2}) - \delta(\omega - \frac{\pi}{2})$

$Y(\omega) = H(\omega)X(\omega)$

$y[n] = |H(0) + H(\frac{\pi}{4})| \sin(\frac{\pi}{4}n + \angle H(\frac{\pi}{4})) + |H(\frac{\pi}{2})| \sin(\frac{\pi}{2}n + \angle H\frac{\pi}{2})$

$H(0) = \frac{1.9}{1+0.9} = 1$ $\quad H(\frac{\pi}{4}) = \frac{1.9e^{j\frac{\pi}{4}}}{e^{j\frac{\pi}{4}}+0.9} = 1.08\angle 21°$ $\quad H(\frac{\pi}{2}) = \frac{1.9e^{j\frac{\pi}{2}}}{e^{j\frac{\pi}{2}}+0.9} = 1.41\angle 42°$

$H(\frac{\pi}{4}) = \frac{1.9(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4})}{\cos(\frac{\pi}{4} + j\sin\frac{\pi}{4} + .9)} = \frac{1.9(.707 + j.707)}{1.607 + j.707} \rightarrow$

$\frac{1.34 + j1.34}{1.607 + j.707} = 1.079\angle 21.25° \rightarrow 21.25° = .37\,rad$

$H(\frac{\pi}{2}) = \frac{1.9(\cos\frac{\pi}{2} + j\sin\frac{\pi}{2})}{\cos\frac{\pi}{2} + j\sin\frac{\pi}{2} + 0.9} = 1.41\angle 41.987° \rightarrow .733\,rad$

$$\boxed{y[n] = 1 + 1.08\sin(\frac{\pi}{4}n + .37\,rad) + 1.41\sin(\frac{\pi}{2}n + \angle H(\frac{\pi}{2}))}$$

The solution was not so bad once I got it setup and realized I had to find H(π/4) and H(π /2). By using equation (5.63)

$$Y(\Omega) = \sum_{k=-\infty}^{\infty} A\pi\left[H(-\Omega_0 + 2\pi k)e^{-j\theta}\delta(\Omega + \Omega_0 - 2\pi k) + H(\Omega_0 + 2\pi k)e^{j\theta}\delta(\Omega - \Omega_0 - 2\pi k)\right]$$

to get a start. In addition, since h[n] is real valued, $|H(-\Omega)|=|H(\Omega)|$ and $\angle H(-\Omega) = -\angle H(\Omega)$, and thus the polar forms of $H(-\Omega_0)$ and $H(\Omega_0)$ are given by

$$H(-\Omega_0) = |H(\Omega_0)|e^{-j\angle H(\Omega_0)} \_and\_ H(\Omega_0) = |H(\Omega_0)|e^{j\angle H(\Omega_0)}$$

Then equation (5.63) above, after some further deduction becomes
$y[n] = A\,|\,H(\Omega_0)\,|\cos(\Omega_0 n + \theta + \angle H(\Omega_0)), \quad n = 0, \pm 1, \pm 2...$

I know the equation uses cos, however I could only get the solution to come out right using sin.

Matlab Code:

```
>> n=0:1:30;
>> x=1+sin((pi/4)*n)+sin((pi/2)*n);
>> for k=1:16;
>> for k=1:16;
h(k)=1.9*(-.9)^(k-1);
end;
>> z=conv(x,h);
>> stem(z)
>> y=1+1.08*sin((pi/4)*n+.37)+1.41*sin((pi/2)*n+.73);
>> stem(y)
>> stem(y)
>> hold
Current plot held
>> stem(z)
>> stem(z)
>> stem(y)
```

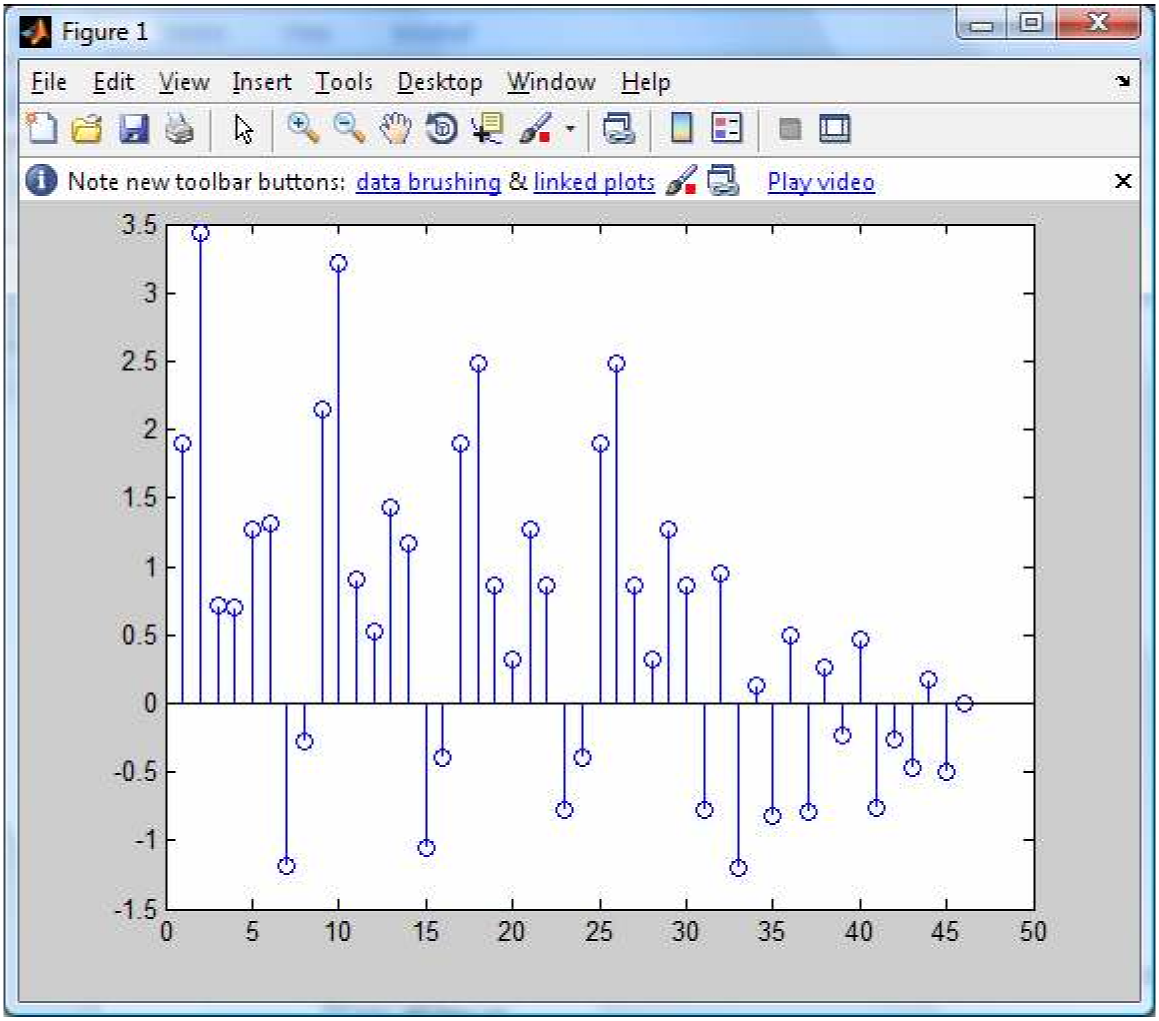

Figure 1: Plot of stem(y)

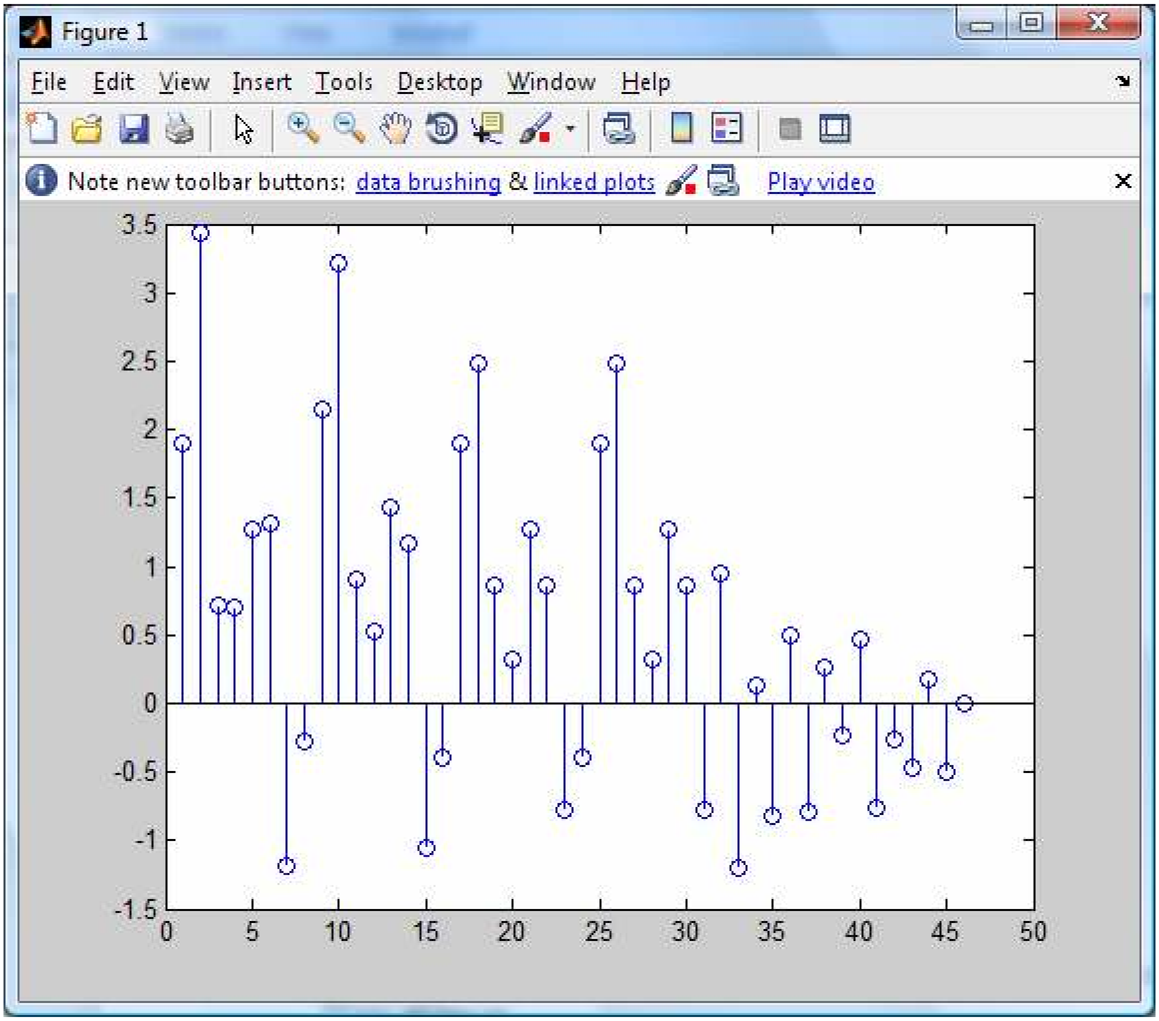

Figure 2: plot of stem(z)

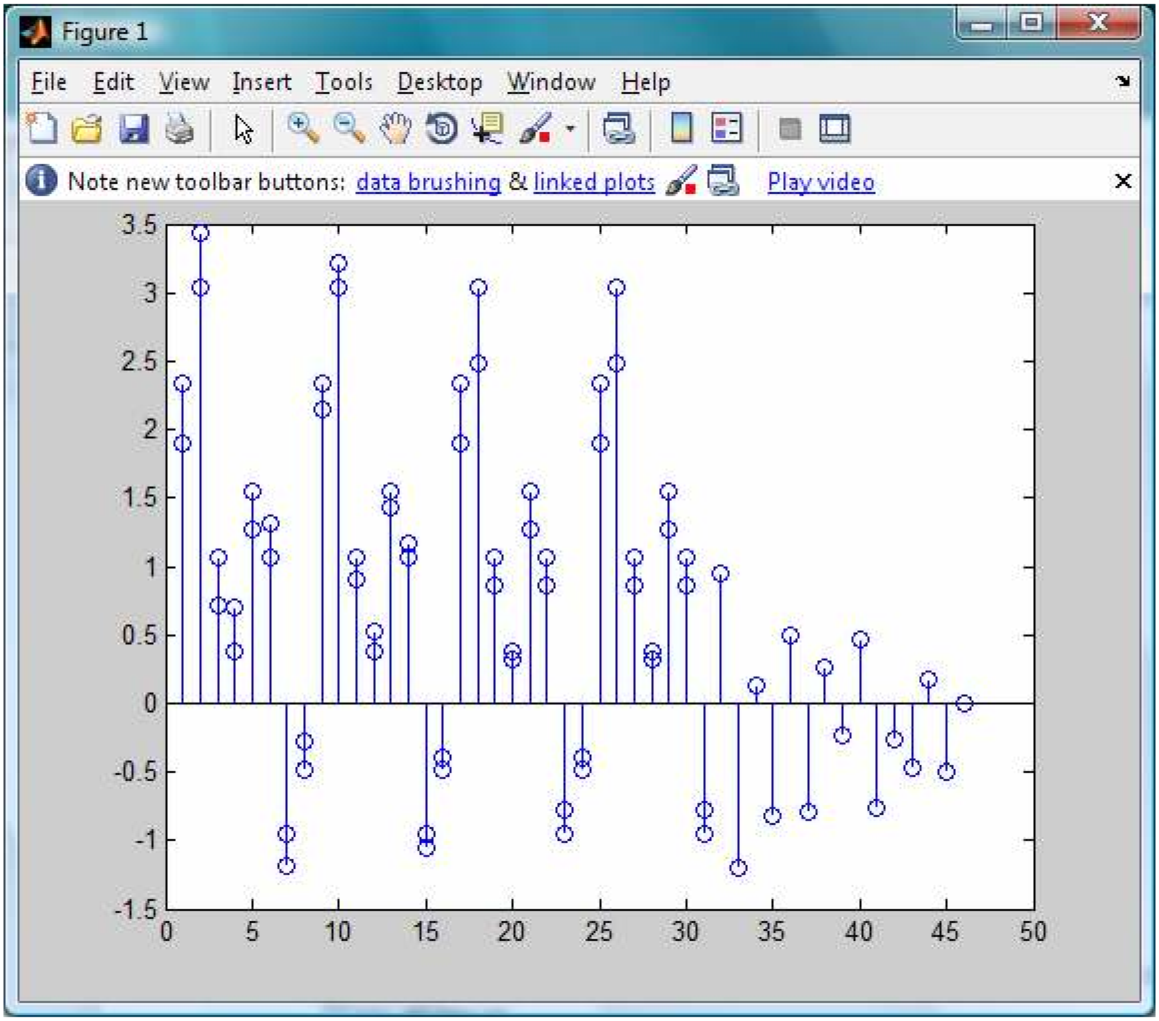

Figure 3: plot of the convolution of stem(y) & stem(z)

I realize that the convolution plot is off by a fraction of a decimal but this was as close as I was able to get it.