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Signals and Systems

Problem 5.45

5.45 a) $y[n+1] + .9y[n] = 1.9x[n+1] \Rightarrow h[n+1] + .9h[n] = 1.9\delta[n+1]$

$h[n] = 1.9(-.9)^{n+1} u[n+1] + .9(1.9)(-.9)^n u[n] = 1.9(-.9)^{n+1}[u[n+1] - u[n]]$

$h[n] = 1.9(-.9)^n u[n]$

$n = -1$

so, $(-.9)^{n+1} = 1$

using $u[n+1] - u[n] = \delta[n+1]$

$= 1.9(.9)^{n+1} [u[n+1] - u[n]]$

$h[n] = 1.9\delta[n+1]$

b)

$H(\omega) = \dfrac{1.9 e^{j\omega}}{e^{j\omega} + .9}$

c) $x(\omega) = 2\pi\delta(\omega) + 5\pi[\delta(\omega + \pi/4) - \delta(\omega - \pi/4)] + 4\pi[\delta(\omega + \pi/2) - \delta(\omega - \pi/2)]$

$y(\omega) = H(\omega)x(\omega)$

$y[n] = |H(n) + H(\pi/4)| \sin(\pi/4 n + \angle H(\pi/4)) + |H(\pi/2)| \sin(\pi/2 n) + \angle H(\pi/2)|$

$H(0) = \dfrac{1.9}{1 + .9} = 1$

$H(\pi/4) = \dfrac{1.9 e^{j\pi/4}}{|e^{j\pi/4} + .9}$
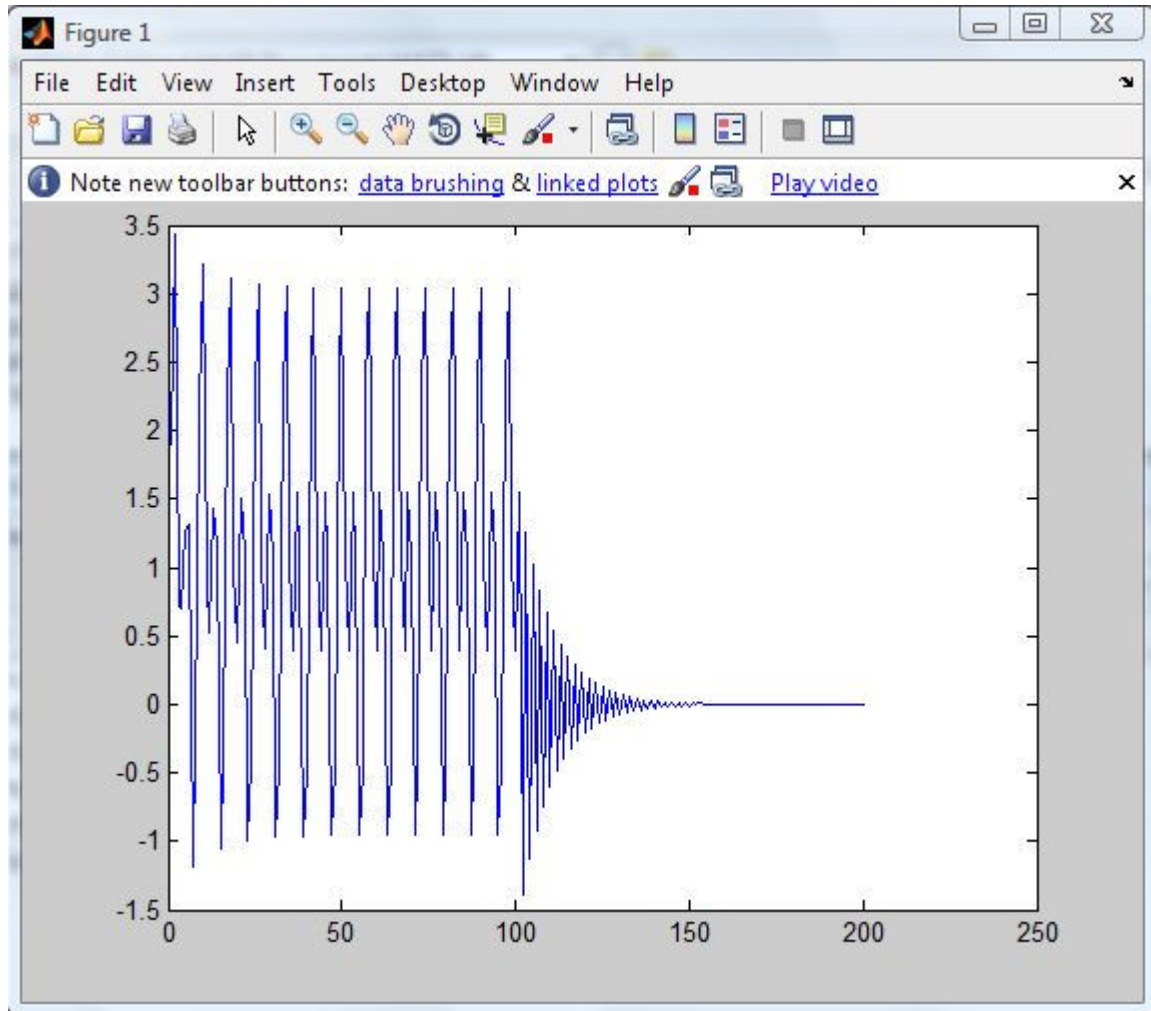
$H(\pi/2) = \dfrac{1.9 e^{j\pi/2}}{|e^{j\pi/2} + .9}$

$H(\pi/4) = \dfrac{1.9[\cos(\pi/4) + j\sin(\pi/4)]}{\cos(\pi/4) + j\sin(\pi/4) + .9} = \dfrac{1.34 + j1.34}{1.6 + j.707} = \dfrac{1.08}{.37} = \angle H(\pi/4)$ ↙ $H(\omega)$

$H(\pi/2) = \dfrac{1.9[\cos(\pi/2) + j\sin(\pi/2)]}{\cos(\pi/2) + j\sin(\pi/4) + .9} = \dfrac{1.9(j)}{j + .9} = \dfrac{1.4}{.733} = \angle H(\pi/2)$ ↖ $H(\omega)$

so

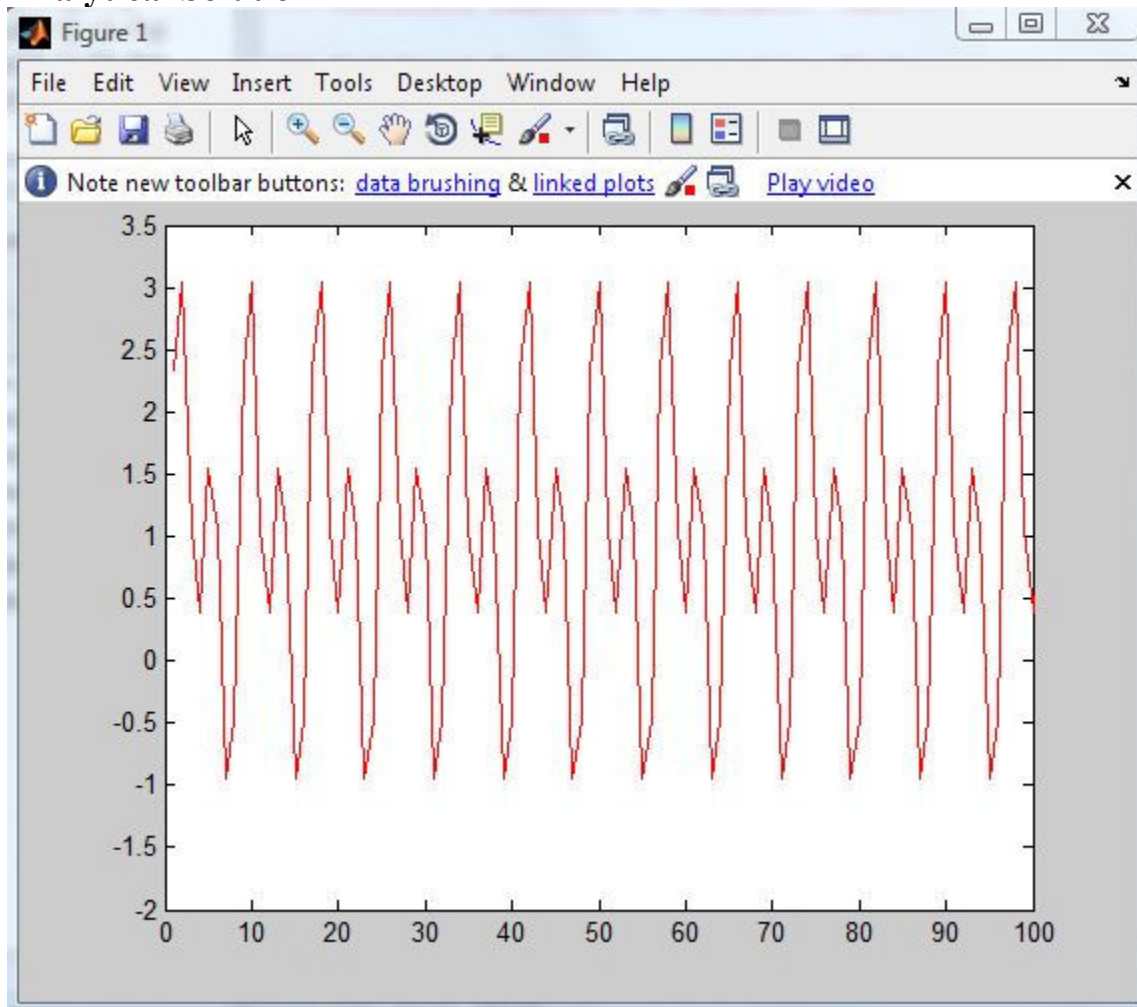$y[n] = |1 + 1.08 \sin(\pi/4 n + .37)| + 1.41 \sin(\pi/2 n + .733)$

Convoluted Signal

Matlab Code
```
n=(0:1:100);
x=1+sin((pi/4)*n)+sin((pi/2)*n);
h=1.9*(-.9).^n;
y=conv(x,h);
z=1+1.08*sin((pi/4)*n+0.37)+1.41*sin((pi/2)*n+0.733);
plot(y,'b');
```

Analytical Solution

Matlab Code
```
n=(0:1:100);
x=1+sin((pi/4)*n)+sin((pi/2)*n);
h=1.9*(-.9).^n;
y=conv(x,h);
z=1+1.08*sin((pi/4)*n+0.37)+1.41*sin((pi/2)*n+0.733);
plot(z,'r');
```

The output response y[n] which results from the input response x[n] is given by the equation

$$y[n] = x[n] \times h[n] \qquad (1)$$

where h[n] is the unit-pulse response. The product of the DTFT's of the two signals h[ω] and x[ω] is equal to the DTFT of a convolution of the two signal h[n] and x[n]. If you take the DTFT of the two sides, this gives the equation

$$Y(\omega) = X(\omega) \times H(\omega) \qquad (2)$$

where X(ω) is the DTFT of the input x[n] and Y(ω) is the DTFT of the output y[n]. H(ω) is the frequency response function of the system. The DTFT of the unit-pulse response h[n] is equal to the frequency response of the system. The magnitude |H(ω)| is the magnitude function of the system and <H(ω) is the phase function of the system.

The analytical solution involved taking the DTFT of the input signal x[n] and the unit-pulse response h[n]. This allows me to multiply the DTFT of the input signal X(ω) and the frequency response H(ω) to get the DTFT Y(ω). Then, I took the inverse DTFT of Y(ω) to get y[n].

Using MATLAB made the math really easy. I took the input signal x[n] and the unit-pulse response h[n] and convoluted the signals together. Also, I plotted the final analytical signal in MATLAB. By comparing the pictures, you can see that the pictures are almost identical except for the decay on the convoluted signal. The decay is caused by the ending of the signal.