Ashley Stough Extra Credit for Test 2 03/27/09
abs90 5.45(c)

Problem 5.45(c) states:
Consider the discrete time system given by the input/output difference equation

$$y[n+1]+0.9y[n]=1.9x[n+1]$$

Compute the output response y[n] to an input of

$$x[n]=1+\sin(\frac{\pi n}{4})+\sin(\frac{\pi n}{2})$$

To work this problem, we need some information from parts a and b of 5.45. First off, in part a, it is proven that

$$h[n]=1.9(-0.9)^n u[n]$$

In part b, we computed the frequency response

$$H(\omega)=\frac{1.9e^{j\omega}}{e^{j\omega}+0.9}$$

So, when we get to part c, we can see that x[n] is a linear equation that can be broken down to smaller, more easily manageable pieces.

$$x_1[n]=1$$

$$x_2[n]=\sin(\frac{\pi n}{4})$$

$$x_3[n]=\sin(\frac{\pi n}{2})$$

We then use our Common DTFT pairs to calculate X(ω) for the three x[n] equations.

$$X_1(\omega)=\sum_{k=-\infty}^{\infty}2\pi\delta(\omega-2\pi k)$$

$$X_2(\omega)=\sum_{k=-\infty}^{\infty} j\pi[\delta(\omega+\frac{\pi}{4}-2\pi k)-\delta(\omega-\frac{\pi}{4}-2\pi k)]$$

$$X_3(\omega)=\sum_{k=-\infty}^{\infty} j\pi[\delta(\omega+\frac{\pi}{2}-2\pi k)-\delta(\omega-\frac{\pi}{2}-2\pi k)]$$

Then we find our Y(ω) by using our different X(ω)s multiplied with H(ω). After we have found this, we simplify the equations using several properties found on page 250 in our textbook. The equations used in my solutions below are:

$$H(\omega)\delta(\omega+c)=H(-\omega)\delta(\omega+c)$$

$$|H(\omega)|=|H(-\omega)|$$

and

$$H(-\omega_0)=|H(\omega_0)|e^{-j\angle H(\omega_0)}$$

$$H(\omega_0)=|H(\omega_0)|e^{j\angle H(\omega_0)}$$

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The first equation is used to find what value to plug into H(ω) in all three sections of Y(ω). The second one is used in the second and third sections of Y(ω) to help simplify the expressions so that we can use our inverse DTFT pairs. Below is the solution to the problem, followed by Matlab code and graphs that support the solution.

$$Y_1(\omega) = H(\omega)X_1(\omega)$$
$$= \sum_{k=-\infty}^{\infty} 2\pi H(\omega)\delta(\omega - 2\pi k)$$
$$= \sum_{k=-\infty}^{\infty} 2\pi H(0)\delta(\omega - 2\pi k)$$
$$= \sum_{k=-\infty}^{\infty} 2\pi \delta(\omega - 2\pi k)$$
Therefore,
$$y_1[n] = 1$$

$$Y_2(\omega) = H(\omega)X_2(\omega)$$
$$= \sum_{k=-\infty}^{\infty} j\pi[H(\omega)\delta(\omega + \frac{\pi}{4} - 2\pi k) - H(\omega)\delta(\omega - \frac{\pi}{4} - 2\pi k)]$$
$$= \sum_{k=-\infty}^{\infty} j\pi[H(\frac{-\pi}{4})\delta(\omega + \frac{\pi}{4} - 2\pi k) - H(\frac{\pi}{4})\delta(\omega - \frac{\pi}{4} - 2\pi k)]$$
$$= \sum_{k=-\infty}^{\infty} j\pi[|H(\frac{-\pi}{4})| e^{-j\angle H(\frac{\pi}{4})}\delta(\omega + \frac{\pi}{4} - 2\pi k) - |H(\frac{\pi}{4})| e^{j\angle H(\frac{\pi}{4})}\delta(\omega - \frac{\pi}{4} - 2\pi k)]$$
$$= |H(\frac{\pi}{4})| \sum_{k=-\infty}^{\infty} j\pi[e^{-j\angle H(\frac{\pi}{4})}\delta(\omega + \frac{\pi}{4} - 2\pi k) - e^{j\angle H(\frac{\pi}{4})}\delta(\omega - \frac{\pi}{4} - 2\pi k)]$$
Therefore,
$$y_2[n] = |H(\frac{\pi}{4})| \sin(\frac{\pi n}{4} + \angle H(\frac{\pi}{4}))$$
$$= 1.082\sin(\frac{\pi n}{4} + 0.371)$$

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$$Y_3(\omega) = H(\omega)X_3(\omega)$$

$$= \sum_{k=-\infty}^{\infty} j\pi[H(\omega)\delta(\omega+\frac{\pi}{2}-2\pi k) - H(\omega)\delta(\omega-\frac{\pi}{2}-2\pi k)]$$

$$= \sum_{k=-\infty}^{\infty} j\pi[H(\frac{-\pi}{2})\delta(\omega+\frac{\pi}{2}-2\pi k) - H(\frac{\pi}{2})\delta(\omega-\frac{\pi}{2}-2\pi k)]$$

$$= \sum_{k=-\infty}^{\infty} j\pi[|H(\frac{-\pi}{2})|e^{-j\angle H(\frac{\pi}{2})}\delta(\omega+\frac{\pi}{2}-2\pi k) - |H(\frac{\pi}{2})|e^{j\angle H(\frac{\pi}{2})}\delta(\omega-\frac{\pi}{2}-2\pi k)]$$

$$= |H(\frac{\pi}{2})| \sum_{k=-\infty}^{\infty} j\pi[e^{-j\angle H(\frac{\pi}{2})}\delta(\omega+\frac{\pi}{2}-2\pi k) - e^{j\angle H(\frac{\pi}{2})}\delta(\omega-\frac{\pi}{2}-2\pi k)]$$

Therefore,

$$y_3[n] = |H(\frac{\pi}{2})| \sin(\frac{\pi n}{2} + \angle H(\frac{\pi}{2}))$$

$$= 1.414\sin(\frac{\pi n}{2} + 0.733)$$

When you put all three y[n] equations together, you get the output response

$$y[n] = 1 + 1.082\sin(\frac{\pi n}{4} + 0.371) + 1.414\sin(\frac{\pi n}{2} + 0.733)$$

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Matlab Code:

```
>> n=0:0.5:150;
>> x=1+sin((pi*n)/4)+sin((pi*n)/2);
>> for k=1:15;
h(k)=1.9*(-0.9)^(k-1);
end;
>> z=conv(x,h);
>> stem(z)
>> y=1+(1.08214*sin(((pi*n)/4)+0.370902))+(1.41421*sin(((pi*n)/2)+0.732815));
>> stem(y)
>> hold
Current plot held
>> stem(z)
```

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Matlab Pictures:

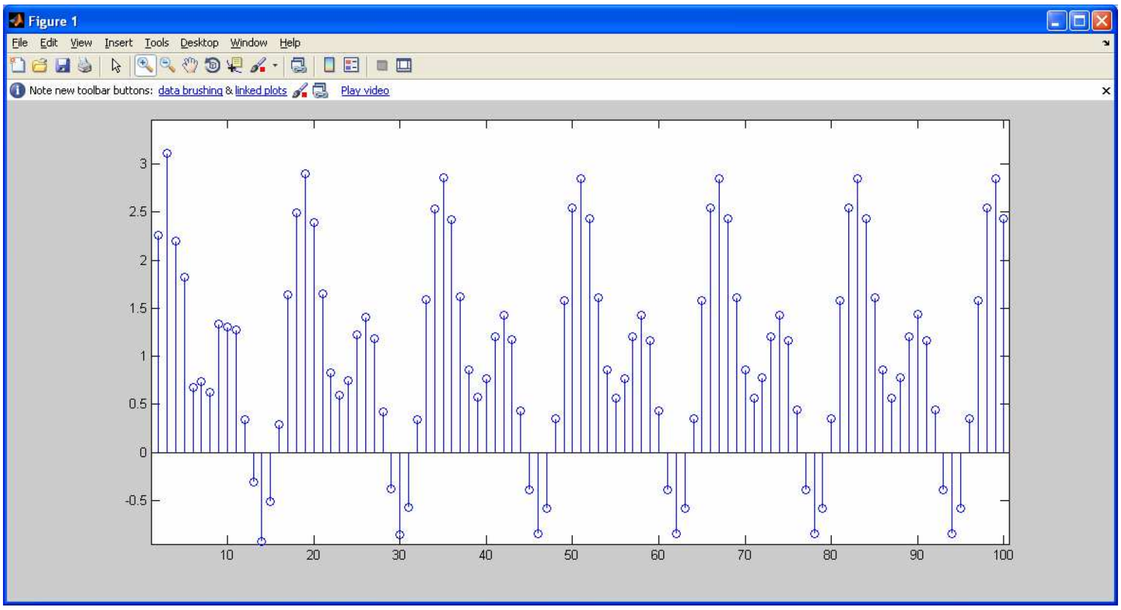

Figure 1: Stem (z)- Matlab's solution

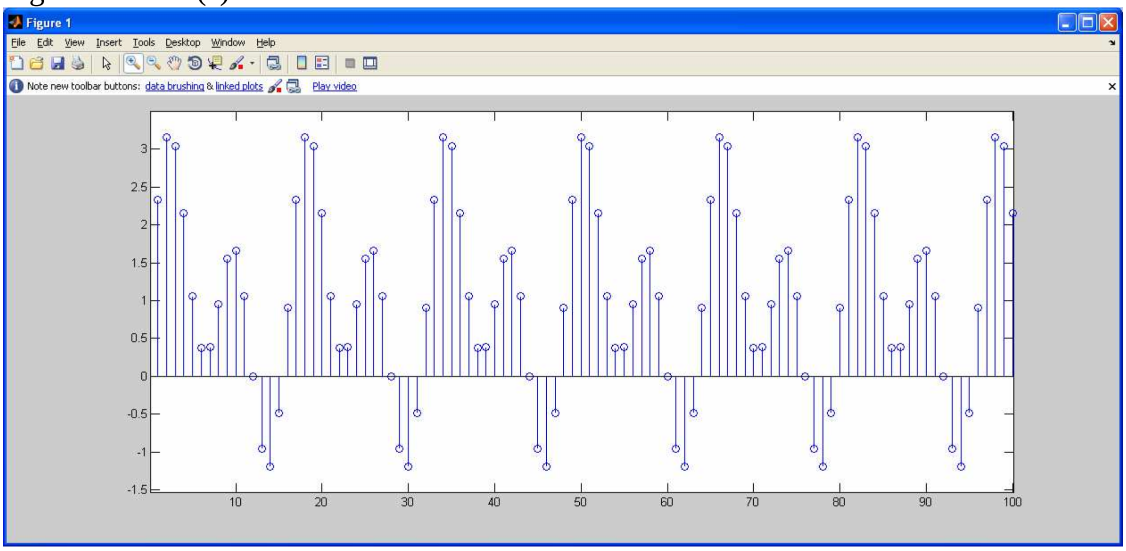

Figure 2: Stem (y)- My solution

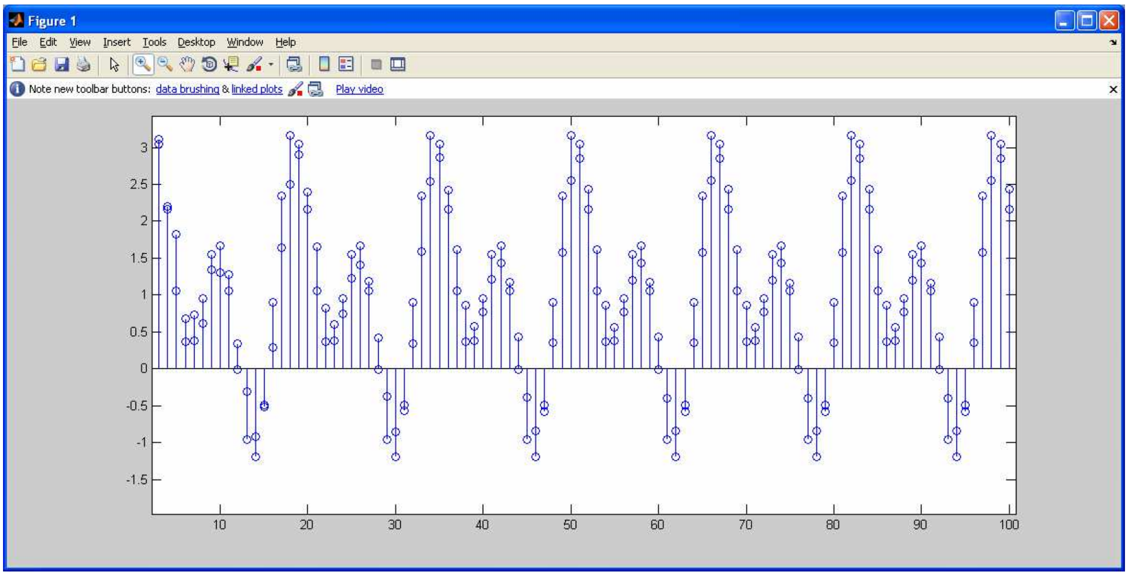

Figure 3: Both Graphs Overlapping With Only Minor Differences