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Extra Credit – 5.45(c)

Part 1: Analytical Solution

Consider the discrete-time system given by the input/output difference equation:

y[n+1]=+0.9y[n]=1.9x[n+1]

The impulse response is given by h[n]=1.9(-0.9)^{n}u[n].

Rewriting the original equation yields:

$$h[n+1]+0.9h[n]=1.9\delta[n+1]$$

Substitute h[n] into the above equation:

$$1.9(-0.9)^{n+1}u[n+1]+0.9[(1.9)(-0.9)^{n}u[n]$$

$$1.9(-0.9)^{n+1}(u(n+1)-u(n))$$

Impulse = 0 for all n, except n=-1

$$(-0.9)^{n+1}\to 1$$

Proves:

$$u(n+1)-u(n)=\delta(n+1)\Rightarrow 1.9\delta[n+1]$$

$$h[n]=1.9(-0.9)^{n}u[n]$$

Compute the output response y[n] to an input of:

$$x[n]=1+\sin(\pi n/4)+\sin(\pi n/2)$$

$$y[n]=|H(0)|+|H(t)|\sin[(t)n+\angle H(t)]+|H(z)|\sin[(z)n+\angle H(z)$$

Since:

$$h[n]=1.9(-0.9)^{n}u[n]$$

Then:

$$H(\omega)=\frac{1.9}{1+0.9e^{-j\omega}}=\frac{1.9e^{j\omega}}{e^{j\omega}+0.9}$$

$$H(0)=\frac{1.9e^{j0}}{e^{j0}+0.9}=\frac{1.9}{1+0.9}=1$$

$$H\left(\frac{\pi}{4}\right)=\frac{1.9e^{j(\pi/4)}}{e^{j(\pi/4)}+0.9}=\frac{1.9\left(\cos\frac{\pi}{4}+j\sin\frac{\pi}{4}\right)}{\left(\cos\frac{\pi}{4}+j\sin\frac{\pi}{4}\right)+0.9}=\frac{1.9(0.7071+j0.7071)}{(0.7071+j0.7071)+0.9}=\frac{1.9\angle 45^{0}}{1.7558\angle 23.75^{0}}$$

$$=1.082\angle 21.25^{0}=1.082\angle\frac{(21.25)(\pi)}{180}rad=1.802\angle 0.371rad$$

$$H\left(\frac{\pi}{2}\right)=\frac{1.9e^{j(\pi/2)}}{e^{j(\pi/2)}+0.9}=\frac{1.9\left(\cos\frac{\pi}{2}+j\sin\frac{\pi}{2}\right)}{\left(\cos\frac{\pi}{2}+j\sin\frac{\pi}{2}\right)+0.9}=\frac{1.9(0+j1)}{(0+j1)+0.9}=\frac{j1.9}{0.9+j1}$$

$$=1.41\angle 41.99^{0}=1.41\angle\frac{(41.99)(\pi)}{180}rad=1.41\angle 0.733rad$$

Plug these values back into the equation stated earlier:

$$y[n]=|H(0)|+\left|H\left(\frac{\pi}{4}\right)\right|\sin\left[\left(\frac{\pi}{4}\right)n+\angle H\left(\frac{\pi}{4}\right)\right]+\left|H\left(\frac{\pi}{2}\right)\right|\sin\left[\left(\frac{\pi}{2}\right)n+\angle H\left(\frac{\pi}{2}\right)\right]$$

The answer is then given by:

$$y[n]=1+1.082\sin\left[\frac{\pi}{4}n+0.371\right]+1.41\sin\left[\frac{\pi}{2}n+0.733\right]$$

This answer can be checked in Matlab.

Part 2: Matlab Code and Graphs

The first part of the code is to show how Matlab can check my calculator values for H:

```
EDU>> syms j;
EDU>> h=[1.9*exp(j*0)]/[exp(j*0)+0.9]

        h =

        1
```

The next part of the code is to obtain the solution graphically using Matlab:

```
EDU>> % Matlab Code for Extra Credit
EDU>> % the first step is to declare 'n' as a range of integers
EDU>> n=0:1:30;
EDU>> % declare the input used to compute the output response
EDU>> x=1+sin((pi/4)*n)+sin((pi/2)*n);
EDU>> % formulate a for loop in order to loop a range of values through the impulse response
EDU>> for s=1:30;
        h(s)=1.9*(-.9)^(s-1);
        end;
EDU>> % plot the input equation in a discrete plot
EDU>> stem(x)
EDU>> % plot the impulse response in a discrete plot
EDU>> stem(h)
EDU>> % convolute the two equations above and name them a variable for easy access
EDU>> c=conv(x,h);
EDU>> % plot the convolution in a discrete plot
EDU>> stem(c)
EDU>> % analytical solution obtained from calculations
EDU>> y=1+1.082*sin((pi/4)*n+0.371)+1.41*sin((pi/2)*n+0.733);
EDU>> % plot the solution in a discrete plot
EDU>> stem(y)
EDU>> % use the hold function to display both graphs on the same plot (convolution and analytical)
EDU>> hold
Current plot held
EDU>> stem(c)
```

Figure 3 displays the convolution of the impulse function and input equation from figures 1 and 2. Figure 4 displays the analytical solution. Both of the graphs were plotted together in figure 5 to compare. There are very small differences in the magnitudes of the points. These differences could be contributed to a small error in calculations. Furthermore, the x axis is different for each plot. Figure 3 only goes to 35 while figure 4 goes to 60. When trying to change these values, errors started occurring so I decided to leave it like this. Basically, Matlab confirmed that my analytical calculation was correct.

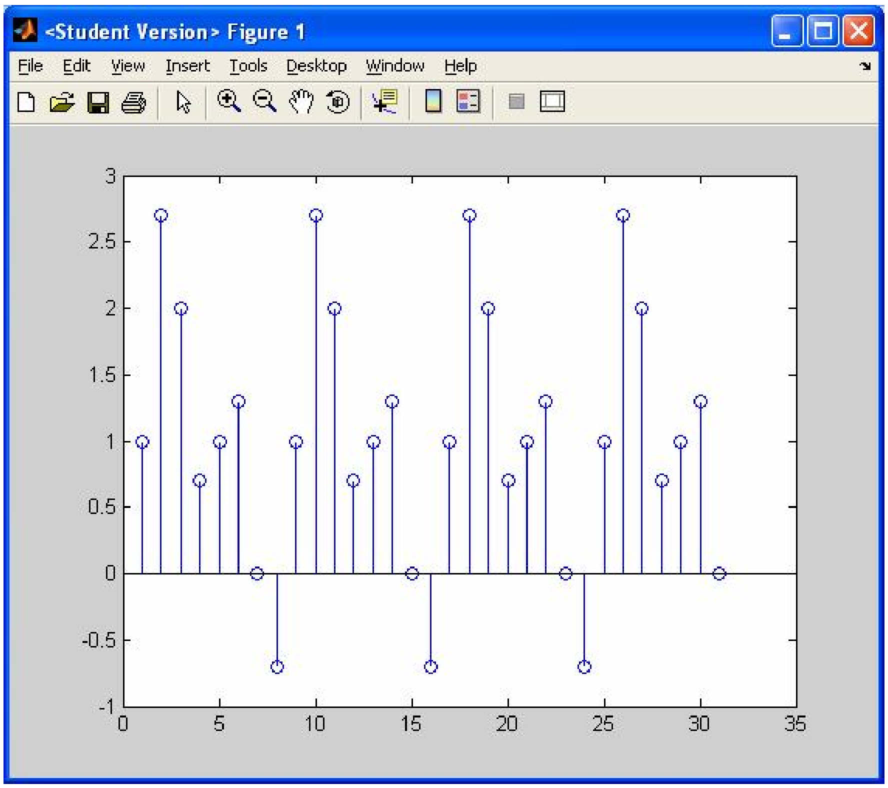

Figure 1: Input Equation: x=1+sin((pi/4)*n)+sin((pi/2)*n)

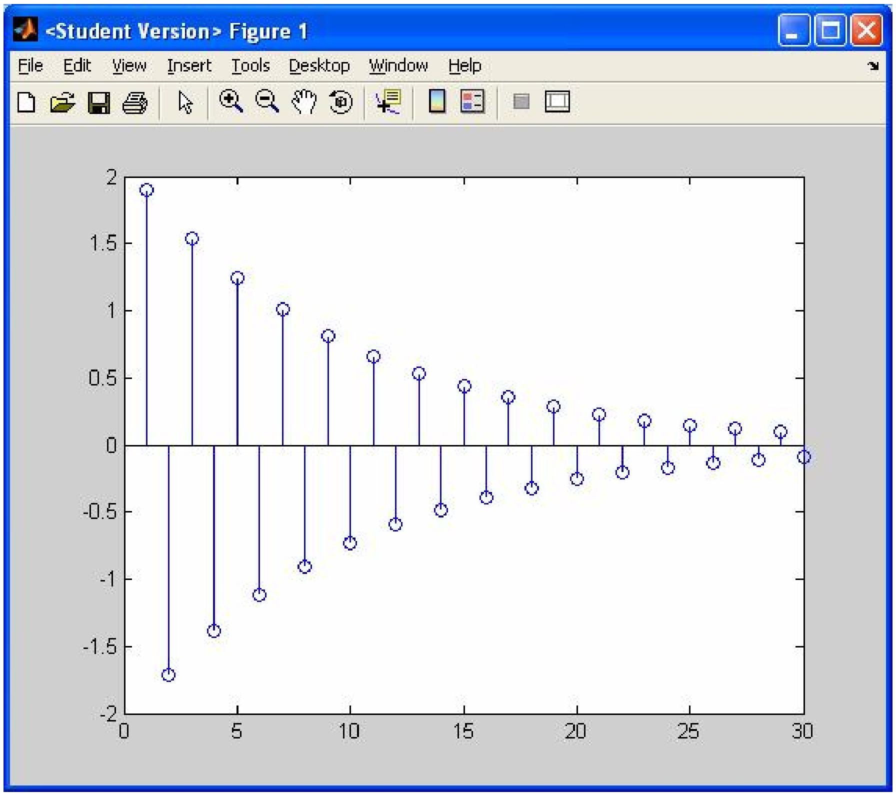

Figure 2: Impulse Response: h[n]=1.9*(-.9)^(n)*u[n]

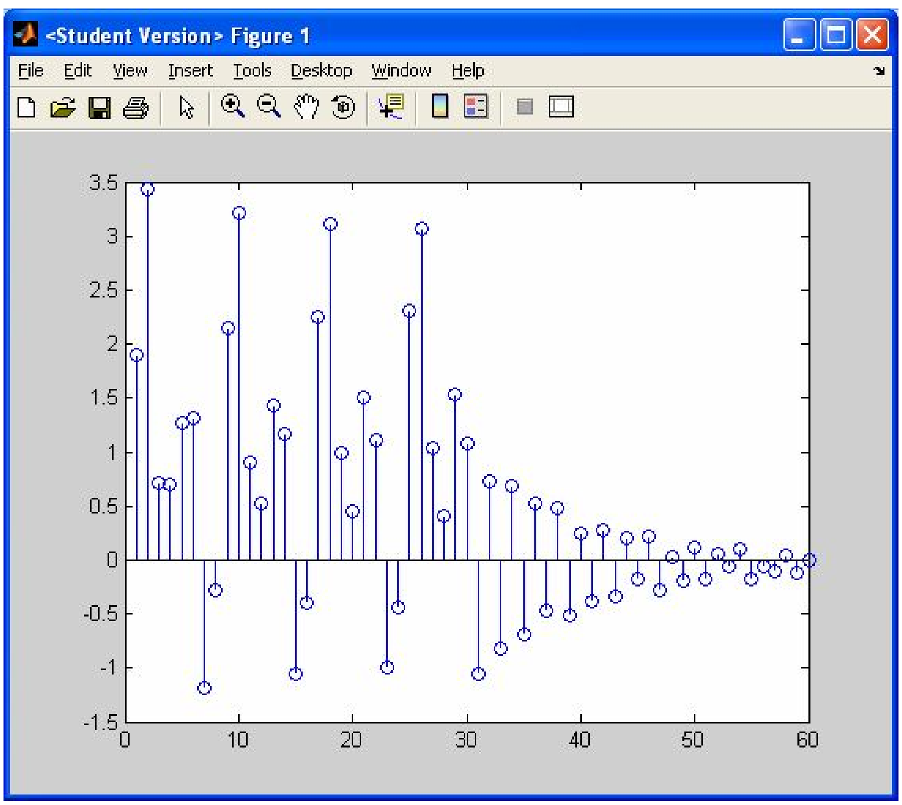

Figure 3: Convolution of the Impulse Response and Input Equation

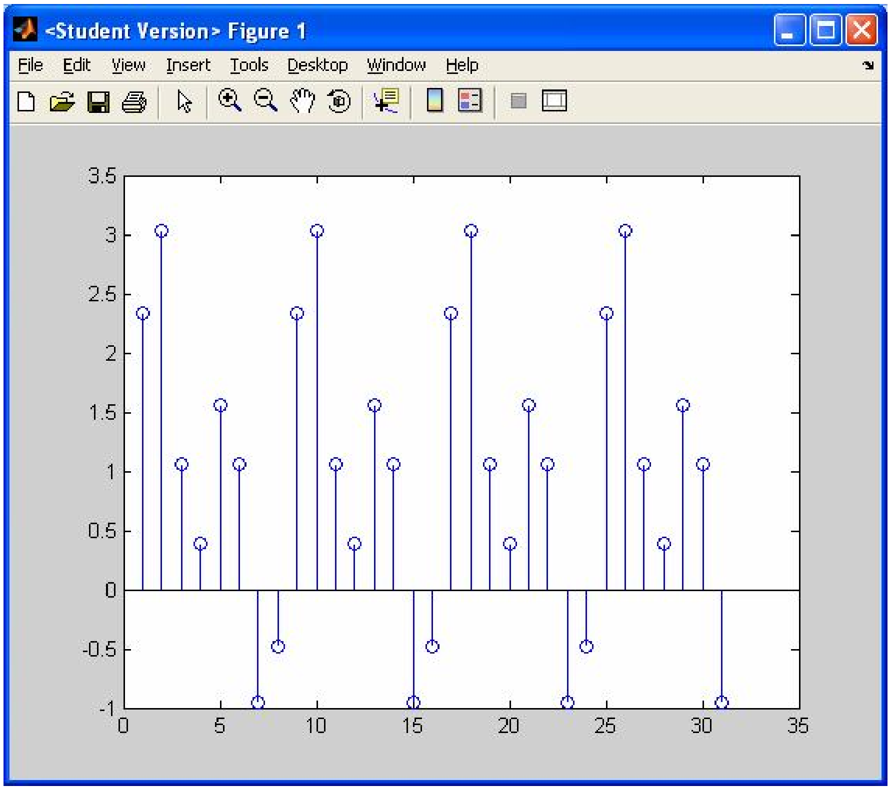

Figure 4: Analytical Solution y[n]

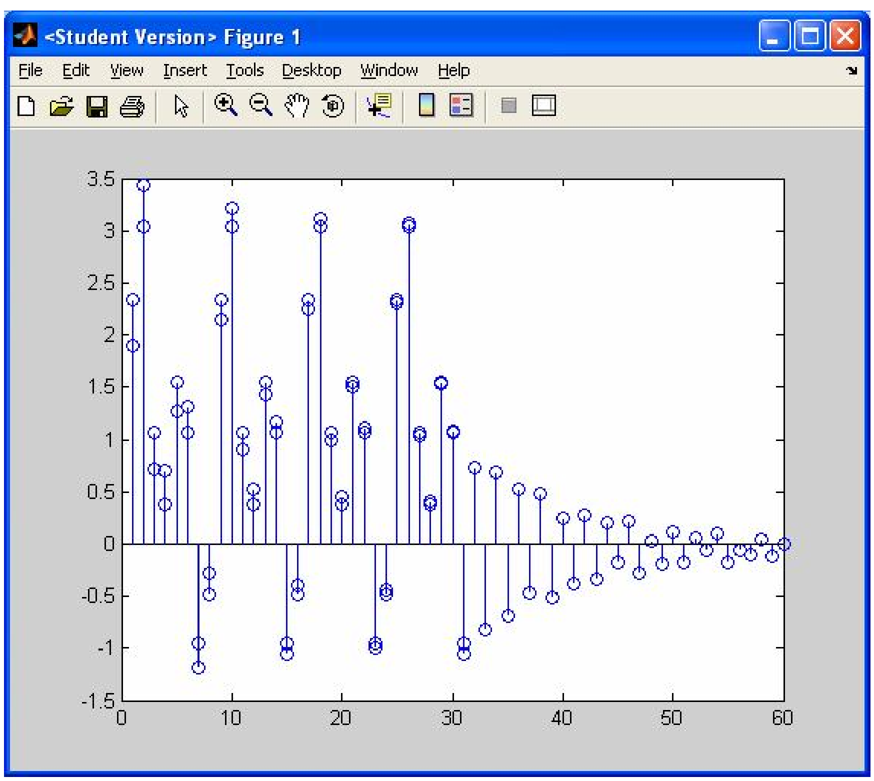

Figure 5: Analytical Solution from Figure 4 and Convolution from Figure 3