

ECE-3161: Signals and Systems  
 Rework 5.45(c) Exam II  
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$$x[n] = 1 + \sin\left(\frac{\pi n}{4}\right) + \sin\left(\frac{\pi n}{2}\right)$$

$$x_1[n] = 1$$

$$X_1(\omega) = \sum_{k=-\infty}^{\infty} 2\pi \delta(\omega - 2\pi k)$$

$$Y_1(\omega) = X_1(\omega)H(\omega) = \sum_{k=-\infty}^{\infty} 2\pi H(\omega) \delta(\omega - 2\pi k) = \sum_{k=-\infty}^{\infty} 2\pi H(0) \delta(\omega - 2\pi k)$$

$$y[n] = 1$$

$$h[n] = 1.9(-0.9)^n u[n] \rightarrow H(\omega) = \frac{1.9 e^{j\omega}}{e^{j\omega} + 0.9} \xrightarrow{\text{DTFT}}$$

$$x_2[n] = \sin\left(\frac{\pi n}{4}\right)$$

$$X_2(\omega) = \sum_{k=-\infty}^{\infty} j\pi \left[ \delta\left(\omega + \frac{\pi}{4} - 2\pi k\right) - \delta\left(\omega - \frac{\pi}{4} - 2\pi k\right) \right]$$

$$Y_2(\omega) = X_2(\omega)H(\omega) = \sum_{k=-\infty}^{\infty} j\pi \left[ H(\omega) \delta\left(\omega + \frac{\pi}{4} - 2\pi k\right) - H(\omega) \delta\left(\omega - \frac{\pi}{4} - 2\pi k\right) \right]$$

$$= \sum_{k=-\infty}^{\infty} j\pi \left[ H\left(\frac{\pi}{4}\right) \delta\left(\omega + \frac{\pi}{4} - 2\pi k\right) - H\left(\frac{\pi}{4}\right) \delta\left(\omega - \frac{\pi}{4} - 2\pi k\right) \right]$$

$$= \sum_{k=-\infty}^{\infty} j\pi \left[ \left| H\left(\frac{\pi}{4}\right) \right| e^{j\angle H\left(\frac{\pi}{4}\right)} \delta\left(\omega + \frac{\pi}{4} - 2\pi k\right) - \left| H\left(\frac{\pi}{4}\right) \right| e^{j\angle H\left(\frac{\pi}{4}\right)} \delta\left(\omega - \frac{\pi}{4} - 2\pi k\right) \right]$$

$$Y_2[n] = \left| H\left(\frac{\pi}{4}\right) \right| \sin\left(\frac{\pi}{4}n + \angle H\left(\frac{\pi}{4}\right)\right)$$

$$1.08214 \sin\left(\frac{\pi}{4}n + 0.370902\right)$$

$$x_3[n] = \sin\left(\frac{\pi n}{2}\right) \quad \omega = \frac{\pi}{2}$$

$$X_3(\omega) = \sum_{k=-\infty}^{\infty} j\pi \left[ \delta\left(\omega + \frac{\pi}{2} - 2\pi k\right) - \delta\left(\omega - \frac{\pi}{2} - 2\pi k\right) \right] \quad \theta = \angle H\left(\frac{\pi}{2}\right)$$

$$Y_3(\omega) = X_3(\omega)H(\omega) = \sum_{k=-\infty}^{\infty} j\pi \left[ H(\omega) \delta\left(\omega + \frac{\pi}{2} - 2\pi k\right) - H(\omega) \delta\left(\omega - \frac{\pi}{2} - 2\pi k\right) \right] \quad \omega = \frac{\pi}{2}$$

$$= \sum_{k=-\infty}^{\infty} j\pi \left[ \left| H\left(\frac{\pi}{2}\right) \right| e^{-j\angle H\left(\frac{\pi}{2}\right)} \delta\left(\omega + \frac{\pi}{2} - 2\pi k\right) - \left| H\left(\frac{\pi}{2}\right) \right| e^{j\angle H\left(\frac{\pi}{2}\right)} \delta\left(\omega - \frac{\pi}{2} - 2\pi k\right) \right]$$

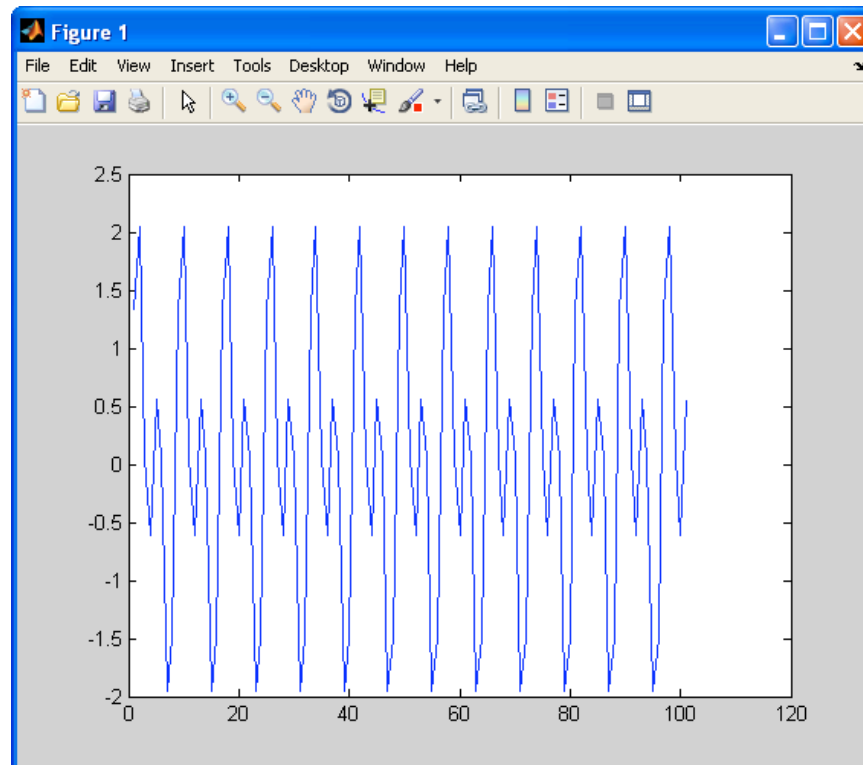
$$y_3[n] = \left| H\left(\frac{\pi}{2}\right) \right| \sin\left(\frac{\pi}{2}n + \angle H\left(\frac{\pi}{2}\right)\right) = 1.41421 \sin\left(\frac{\pi}{2}n + 0.732815\right)$$

$$y[n] = 1 + 1.08214 \sin\left(\frac{\pi}{4}n + 0.370902\right) + 1.41421 \sin\left(\frac{\pi}{2}n + 0.732815\right)$$

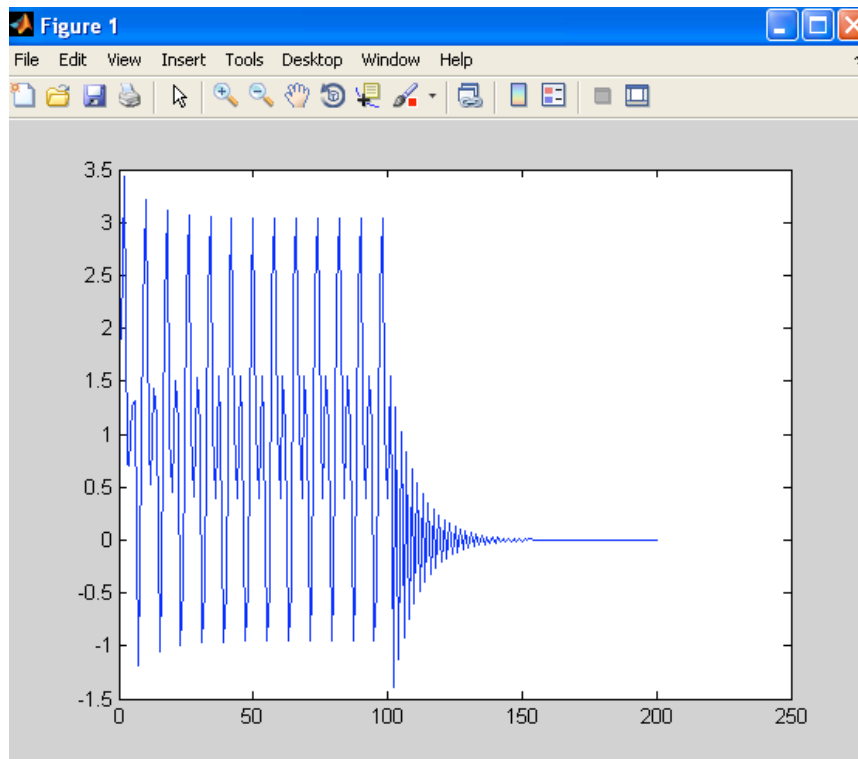
## Command Window

```
>> n=(0:1:100);  
>> x=1+sin(pi*n/4)+sin(pi*n/2);  
>> h=1.9*(-.9).^n;  
>> y=conv(x,h);  
>> a=1.08214*sin((pi/4)*n+0.370902)+1.41421*sin((pi/2)*n+0.732815);  
>> plot(a);  
>> plot(y);  
>>
```

**Figure 1:** MATLAB Code



**Figure 1:** Analytical Solution



**Figure 2:** MATLAB Solution

### **Explanation of work:**

To find the output, we needed to convolute the input and the unit pulse response. Converting to a DTFT allows this since convolution in the time domain is the same as multiplication in the frequency domain.  $h[n]$  was also converted to a frequency response function. Using equation (5.60) from pg. 249 of our book, we may calculate the output. Our frequencies were inserted into the frequency response function found previously in the problem.

I was able to verify this in MATLAB by taking the convolution of input  $x[n]$  and the impulse response  $h[n]$ . It may be noted the decay of the signal is about 40, although the analytical leaves out the decay because of its output. It is amazing how much easier MATLAB made all of this work and gave us a graphical representation of the output.