

Name: \_\_\_\_\_

Problem	Points	Score
7.40(b)	15	
7.40(c)	15	
7.40(d)	15	
8.9(a)(i)	15	
8.9(a)(ii)	10	
8.9(a)(iii)	10	
9.1(a)	10	
9.1(c)	10	
Total	100	

Notes:

- (1) The exam is closed books and notes except for one double-sided sheet of notes.
- (2) Please indicate clearly your answer to the problem.
- (3) The details of your solutions are more important than the answers. Please explain your solutions clearly and include as many details as possible.

7.40. For the following linear time-invariant discrete-time systems with unit-pulse response  $h[n]$ , determine if the system is BIBO stable.

(b)  $h[n] = (1/n)u[n-1]$

$$\sum_{n=0}^{\infty} |h[n]| = \sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

So the system is not BIBO stable

(c)  $h[n] = (1/n^2)u[n-1]$

$$\sum_{n=0}^{\infty} |h[n]| = \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty \text{ so the system is BIBO stable}$$

(d)  $h[n] = e^{-n} \sin(\pi n/6)u[n]$

$$\sum_{n=0}^{\infty} |h[n]| = \sum_{n=0}^{\infty} |e^{-n} \sin(\frac{\pi n}{6})| \leq \sum_{n=0}^{\infty} e^{-n} = \frac{1}{1-e^{-1}} < \infty$$

thus the system is BIBO stable

**8.9(a).** Determine if the system is critically damped, underdamped, or overdamped.

(i)  $H(s) = \frac{32}{s^2 + 4s + 16}$

(ii)  $H(s) = \frac{32}{s^2 + 8s + 16}$

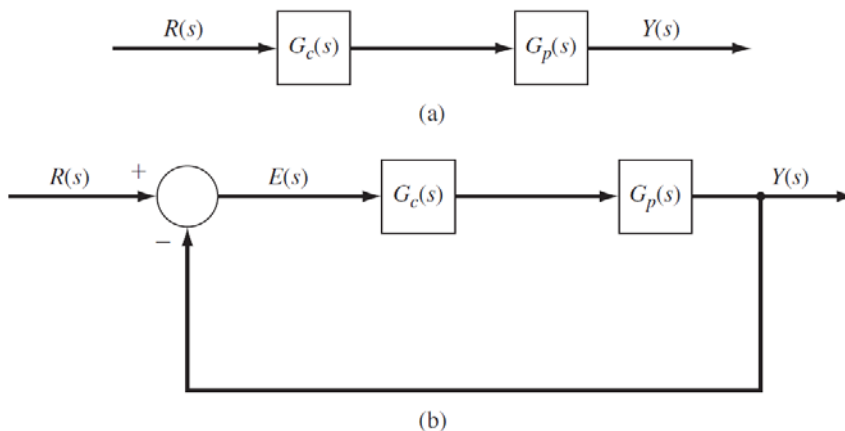
(iii)  $H(s) = \frac{32}{s^2 + 10s + 16}$

- 8.9 (a) i. poles at  $-2 \pm j3.46 \Rightarrow$  underdamped  
ii. poles at  $-4, -4 \Rightarrow$  critically damped  
iii. poles at  $-8, -2 \Rightarrow$  overdamped

9.1. Consider the transfer function:

$$G_p(s) = \frac{1}{s + 0.1}$$

(a) An open-loop control is shown in Figure P9.1(a). Design the control system,  $G_c(s)$ , so that the combined plant and controller,  $G_c(s)G_p(s)$ , has a pole at  $p = -2$ .



$$(a) \quad G_c(s) = \frac{k(s+0.1)}{(s+2)} \quad R(s) = \frac{r_0}{s}$$

$$Y(s) = \frac{k}{(s+2)} R(s) \quad e_{ss} = r_0 - \lim_{s \rightarrow 0} s Y(s)$$

$$e_{ss} = 0 = r_0 - \frac{k r_0}{2} \Rightarrow k = 2$$

$$G_c(s) = \frac{2(s+0.1)}{(s+2)}$$

(c) A feedback controller  $G_c(s) = 2(s+0.1)/s$  is used in place of open-loop control as shown in Figure P9.1b. Verify that the closed-loop pole of the nominal system is at  $p = -2$ .

$$(c) \quad \text{Closed loop transfer function } G_{cl}(s) = \frac{G_c G_p}{1 + G_c G_p}$$

$$G_{cl}(s) = \frac{\frac{2}{s}}{1 + \frac{2}{s}} = \frac{2}{(s+2)}$$

$$\lim_{s \rightarrow 0} s Y(s) = \lim_{s \rightarrow 0} \frac{2 r_0}{(s+2)} = r_0 \Rightarrow e_{ss} = 0$$