TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{array}{l} x(t) \\ y(t) \end{array}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \measuredangle a_k = -\measuredangle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}v\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

TABLE 3.1 Properties of the Fourier Transform

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Right or left shift in time	$x(t - c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X\left(\frac{\omega}{a}\right) \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by a power of t	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \ldots$
Multiplication by a complex exponential	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0$ real
Multiplication by $\sin(\omega_0 t)$	$x(t)\sin(\omega_0 t) \leftrightarrow \frac{j}{2}[X(\omega + \omega_0) - X(\omega - \omega_0)]$
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation in the time domain	$\frac{d^n}{dt^n}x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \ldots$
Integration in the time domain	$\int_{-\infty}^{t} x(\lambda)\, d\lambda \leftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Convolution in the time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in the time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)\, d\omega$
Special case of Parseval's theorem	$\int_{-\infty}^{\infty} x^2(t)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2\, d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

TABLE 3.2 Common Fourier Transform Pairs

$1, -\infty < t < \infty \leftrightarrow 2\pi\delta(\omega)$
$-0.5 + u(t) \leftrightarrow \dfrac{1}{j\omega}$
$u(t) \leftrightarrow \pi\delta(\omega) + \dfrac{1}{j\omega}$
$\delta(t) \leftrightarrow 1$
$\delta(t - c) \leftrightarrow e^{-j\omega c}$, c any real number
$e^{-bt}u(t) \leftrightarrow \dfrac{1}{j\omega + b}, b > 0$
$e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega - \omega_0)$, ω_0 any real number
$p_\tau(t) \leftrightarrow \tau \operatorname{sinc}\dfrac{\tau\omega}{2\pi}$
$\tau \operatorname{sinc}\dfrac{\tau t}{2\pi} \leftrightarrow 2\pi p_\tau(\omega)$
$\left(1 - \dfrac{2\|t\|}{\tau}\right)p_\tau(t) \leftrightarrow \dfrac{\tau}{2}\operatorname{sinc}^2\left(\dfrac{\tau\omega}{4\pi}\right)$
$\dfrac{\tau}{2}\operatorname{sinc}^2\left(\dfrac{\tau t}{4\pi}\right) \leftrightarrow 2\pi\left(1 - \dfrac{2\|\omega\|}{\tau}\right)p_\tau(\omega)$
$\cos(\omega_0 t) \leftrightarrow \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$
$\cos(\omega_0 t + \theta) \leftrightarrow \pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$
$\sin(\omega_0 t) \leftrightarrow j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
$\sin(\omega_0 t + \theta) \leftrightarrow j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$

TABLE 4.2 Properties of the DTFT

Property	Transform Pair/Property
Linearity	$ax[n] + bv[n] \leftrightarrow aX(\Omega) + bV(\Omega)$
Right or left shift in time	$x[n-q] \leftrightarrow X(\Omega)e^{-jq\Omega}$, q any integer
Time reversal	$x[-n] \leftrightarrow X(-\Omega) = \overline{X(\Omega)}$
Multiplication by n	$nx[n] \leftrightarrow j\frac{d}{d\Omega}X(\Omega)$
Multiplication by a complex exponential	$x[n]e^{jn\Omega_0} \leftrightarrow X(\Omega - \Omega_0)$, Ω_0 real
Multiplication by $\sin \Omega_0 n$	$x[n] \sin \Omega_0 n \leftrightarrow \frac{j}{2}[X(\Omega + \Omega_0) - X(\Omega - \Omega_0)]$
Multiplication by $\cos \Omega_0 n$	$x[n] \cos \Omega_0 n \leftrightarrow \frac{1}{2}[X(\Omega + \Omega_0) + X(\Omega - \Omega_0)]$
Convolution in the time domain	$x[n] * v[n] \leftrightarrow X(\Omega)V(\Omega)$
Summation	$\sum_{i=0}^{n} x[i] \leftrightarrow \frac{1}{1 - e^{-j\Omega}}X(\Omega) + \sum_{n=-\infty}^{\infty} \pi X(2\pi n)\delta(\Omega - 2\pi n)$
Multiplication in the time domain	$x[n]v[n] \leftrightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega - \lambda)V(\lambda)\, d\lambda$
Parseval's theorem	$\sum_{n=-\infty}^{\infty} x[n]v[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \overline{X(\Omega)}V(\Omega)\, d\Omega$
Special case of Parseval's theorem	$\sum_{n=-\infty}^{\infty} x^2[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(\Omega)\rvert^2\, d\Omega$
Relationship to inverse CTFT	If $x[n] \leftrightarrow X(\Omega)$ and $\gamma(t) \leftrightarrow X(\omega)p_{2\pi}(\omega)$, then $x[n] = \gamma(t)\rvert_{t=n} = \gamma(n)$

TABLE 4.1 Common DTFT Pairs

$1, \text{all } n \leftrightarrow \sum_{k=-\infty}^{\infty} 2\pi\delta(\Omega - 2\pi k)$

$\text{sgn}[n] \leftrightarrow \dfrac{2}{1 - e^{-j\Omega}}, \text{ where sgn}[n] = \begin{cases} 1, & n = 0, 1, 2, \ldots \\ -1, & n = -1, -2, \ldots \end{cases}$

$u[n] \leftrightarrow \dfrac{1}{1 - e^{-j\Omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\Omega - 2\pi k)$

$\delta[n] \leftrightarrow 1$

$\delta[n - q] \leftrightarrow e^{-jq\Omega}, q = \pm 1, \pm 2, \ldots$

$a^n u[n] \leftrightarrow \dfrac{1}{1 - ae^{-j\Omega}}, \quad |a| < 1$

$e^{j\Omega_0 n} \leftrightarrow \sum_{k=-\infty}^{\infty} 2\pi\delta(\Omega - \Omega_0 - 2\pi k)$

$p[n] \leftrightarrow \dfrac{\sin\left[\left(q + \frac{1}{2}\right)\Omega\right]}{\sin(\Omega/2)}$

$\dfrac{B}{\pi}\text{sinc}\left(\dfrac{B}{\pi}n\right) \leftrightarrow \sum_{k=-\infty}^{\infty} p_{2B}(\Omega + 2\pi k)$

$\cos \Omega_0 n \leftrightarrow \sum_{k=-\infty}^{\infty} \pi[\delta(\Omega + \Omega_0 - 2\pi k) + \delta(\Omega - \Omega_0 - 2\pi k)]$

$\sin \Omega_0 n \leftrightarrow \sum_{k=-\infty}^{\infty} j\pi[\delta(\Omega + \Omega_0 - 2\pi k) - \delta(\Omega - \Omega_0 - 2\pi k)]$

$\cos(\Omega_0 n + \theta) \leftrightarrow \sum_{k=-\infty}^{\infty} \pi[e^{-j\theta}\delta(\Omega + \Omega_0 - 2\pi k) + e^{j\theta}\delta(\Omega - \Omega_0 - 2\pi k)]$

TABLE 6.1 Properties of the Laplace Transform

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(s) + bV(s)$
Right shift in time	$x(t-c)u(t-c) \leftrightarrow e^{-cs}X(s), c > 0$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X\left(\frac{s}{a}\right), a > 0$
Multiplication by a power of t	$t^N x(t) \leftrightarrow (-1)^N \frac{d^N}{ds^N} X(s), N = 1, 2, \ldots$
Multiplication by an exponential	$e^{at}x(t) \leftrightarrow X(s-a)$, a real or complex
Multiplication by sin ωt	$x(t)\sin\omega t \leftrightarrow \frac{j}{2}[X(s+j\omega) - X(s-j\omega)]$
Multiplication by cos ωt	$x(t)\cos\omega t \leftrightarrow \frac{1}{2}[X(s+j\omega) + X(s-j\omega)]$
Differentiation in the time domain	$\dot{x}(t) \leftrightarrow sX(s) - x(0)$
Second derivative	$\ddot{x}(t) \leftrightarrow s^2X(s) - sx(0) - \dot{x}(0)$
Nth derivative	$x^{(N)}(t) \leftrightarrow s^N X(s) - s^{N-1}x(0) - s^{N-2}\dot{x}(0) - \cdots - sx^{(N-2)}(0) - x^{(N-1)}(0)$
Integration	$\int_0^t x(\lambda)\,d\lambda \leftrightarrow \frac{1}{s}X(s)$
Convolution	$x(t) * v(t) \leftrightarrow X(s)V(s)$
Initial-value theorem	$x(0) = \lim_{s\to\infty} sX(s)$
	$\dot{x}(0) = \lim_{s\to\infty} [s^2X(s) - sx(0)]$
	$x^{(N)}(0) = \lim_{s\to\infty} [s^{N+1}X(s) - s^N x(0) - s^{N-1}\dot{x}(0) - \cdots - sx^{(N-1)}(0)]$
Final-value theorem	If $\lim_{t\to\infty} x(t)$ exists, then $\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$

TABLE 6.2 Common Laplace Transform Pairs

$$u(t) \leftrightarrow \frac{1}{s}$$

$$u(t) - u(t-c) \leftrightarrow \frac{1-e^{-cs}}{s}, c > 0$$

$$t^N u(t) \leftrightarrow \frac{N!}{s^{N+1}}, N = 1, 2, 3, \ldots$$

$$\delta(t) \leftrightarrow 1$$

$$\delta(t-c) \leftrightarrow e^{-cs}, c > 0$$

$$e^{-bt}u(t) \leftrightarrow \frac{1}{s+b}, b \text{ real or complex}$$

$$t^N e^{-bt}u(t) \leftrightarrow \frac{N!}{(s+b)^{N+1}}, \quad N = 1, 2, 3, \ldots$$

$$(\cos \omega t)u(t) \leftrightarrow \frac{s}{s^2+\omega^2}$$

$$(\sin \omega t)u(t) \leftrightarrow \frac{\omega}{s^2+\omega^2}$$

$$(\cos^2 \omega t)u(t) \leftrightarrow \frac{s^2+2\omega^2}{s(s^2+4\omega^2)}$$

$$(\sin^2 \omega t)u(t) \leftrightarrow \frac{2\omega^2}{s(s^2+4\omega^2)}$$

$$(e^{-bt}\cos \omega t)u(t) \leftrightarrow \frac{s+b}{(s+b)^2+\omega^2}$$

$$(e^{-bt}\sin \omega t)u(t) \leftrightarrow \frac{\omega}{(s+b)^2+\omega^2}$$

$$(t\cos \omega t)u(t) \leftrightarrow \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$$

$$(t\sin \omega t)u(t) \leftrightarrow \frac{2\omega s}{(s^2+\omega^2)^2}$$

$$(te^{-bt}\cos \omega t)u(t) \leftrightarrow \frac{(s+b)^2-\omega^2}{[(s+b)^2+\omega^2]^2}$$

$$(te^{-bt}\sin \omega t)u(t) \leftrightarrow \frac{2\omega(s+b)}{[(s+b)^2+\omega^2]^2}$$

TABLE 7.2 Properties of the z-Transform

Property	Transform Pair/Property
Linearity	$ax[n] + bv[n] \leftrightarrow aX(z) + bV(z)$
Right shift of $x[n]u[n]$	$x[n-q]u[n-q] \leftrightarrow z^{-q}X(z)$
Right shift of $x[n]$	$x[n-1] \leftrightarrow z^{-1}X(z) + x[-1]$ $x[n-2] \leftrightarrow z^{-2}X(z) + x[-2] + z^{-1}x[-1]$ $\vdots$ $x[n-q] \leftrightarrow z^{-q}X(z) + x[-q] + z^{-1}x[-q+1] + \cdots + z^{-q+1}x[-1]$
Left shift in time	$x[n+1] \leftrightarrow zX(z) - x[0]z$ $x[n+2] \leftrightarrow z^2X(z) - x[0]z^2 - x[1]z$ $x[n+q] \leftrightarrow z^qX(z) - x[0]z^q - x[1]z^{q-1} - \cdots - x[q-1]z$
Multiplication by n	$nx[n] \leftrightarrow -z\dfrac{d}{dz}X(z)$
Multiplication by n^2	$n^2x[n] \leftrightarrow z\dfrac{d}{dz}X(z) + z^2\dfrac{d^2}{dz^2}X(z)$
Multiplication by a^n	$a^nx[n] \leftrightarrow X\left(\dfrac{z}{a}\right)$
Multiplication by $\cos \Omega n$	$(\cos \Omega n)x[n] \leftrightarrow \dfrac{1}{2}[X(e^{j\Omega}z) + X(e^{-j\Omega}z)]$
Multiplication by $\sin \Omega n$	$(\sin \Omega n)x[n] \leftrightarrow \dfrac{j}{2}[X(e^{j\Omega}z) - X(e^{-j\Omega}z)]$
Summation	$\sum_{i=0}^{n} x[i] \leftrightarrow \dfrac{z}{z-1}X(z)$
Convolution	$x[n] * v[n] \leftrightarrow X(z)V(z)$
Initial-value theorem	$x[0] = \lim_{z\to\infty} X(z)$ $x[1] = \lim_{z\to\infty} [zX(z) - zX[0]]$ $\vdots$ $x[q] = \lim_{z\to\infty} [z^qX(z) - z^qx[0] - z^{q-1}x[1] - \cdots - zx[q-1]]$
Final-value theorem	If $X(z)$ is rational and the poles of $(z-1)X(z)$ have magnitudes <1, then $\lim_{n\to\infty} x[n] = [(z-1)X(z)]_{z=1}$

TABLE 7.3 Common z-Transform Pairs

$\delta[n] \leftrightarrow 1$
$\delta[n-q] \leftrightarrow \dfrac{1}{z^q}, \quad q = 1, 2, \ldots$
$u[n] \leftrightarrow \dfrac{z}{z-1}$
$u[n] - u[n-q] \leftrightarrow \dfrac{z^q - 1}{z^{q-1}(z-1)}, \quad q = 1, 2, \ldots$
$a^n u[n] \leftrightarrow \dfrac{z}{z-a}$, a real or complex
$nu[n] \leftrightarrow \dfrac{z}{(z-1)^2}$
$(n+1)u[n] \leftrightarrow \dfrac{z^2}{(z-1)^2}$
$n^2 u[n] \leftrightarrow \dfrac{z(z+1)}{(z-1)^3}$
$na^n u[n] \leftrightarrow \dfrac{az}{(z-a)^2}$
$n^2 a^n u[n] \leftrightarrow \dfrac{az(z+a)}{(z-a)^3}$
$n(n+1)a^n u[n] \leftrightarrow \dfrac{2az^2}{(z-a)^3}$
$(\cos \Omega n)u[n] \leftrightarrow \dfrac{z^2 - (\cos \Omega)z}{z^2 - (2\cos \Omega)z + 1}$
$(\sin \Omega n)u[n] \leftrightarrow \dfrac{(\sin \Omega)z}{z^2 - (2\cos \Omega)z + 1}$
$a^n(\cos \Omega n)u[n] \leftrightarrow \dfrac{z^2 - (a\cos \Omega)z}{z^2 - (2a\cos \Omega)z + a^2}$
$a^n(\sin \Omega n)u[n] \leftrightarrow \dfrac{(a\sin \Omega)z}{z^2 - (2a\cos \Omega)z + a^2}$