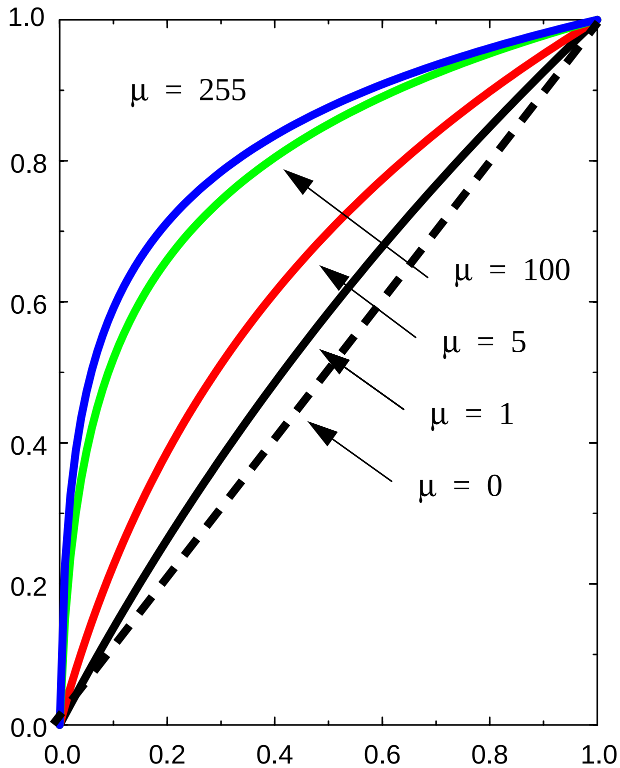
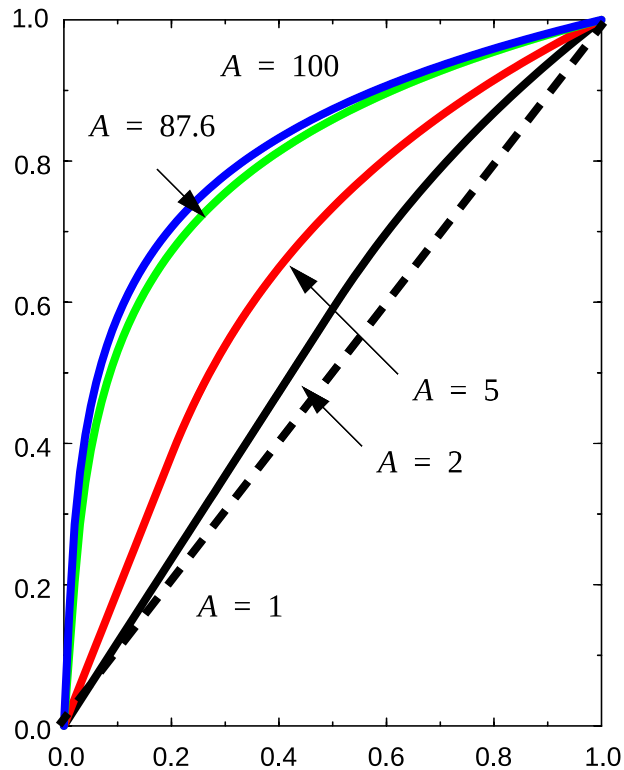
PDF-Optimized Quantizers

- Shape the quantizer to the statistics of the input signal so that all quantization levels are equally important
- Exploit logarithmic sensitivity of human hearing
- Need simple solution:

$$y = \frac{\log(1+\mu x)}{\log(1+\mu)} \qquad \text{for } x \ge 0 \qquad \mu\text{-law}$$

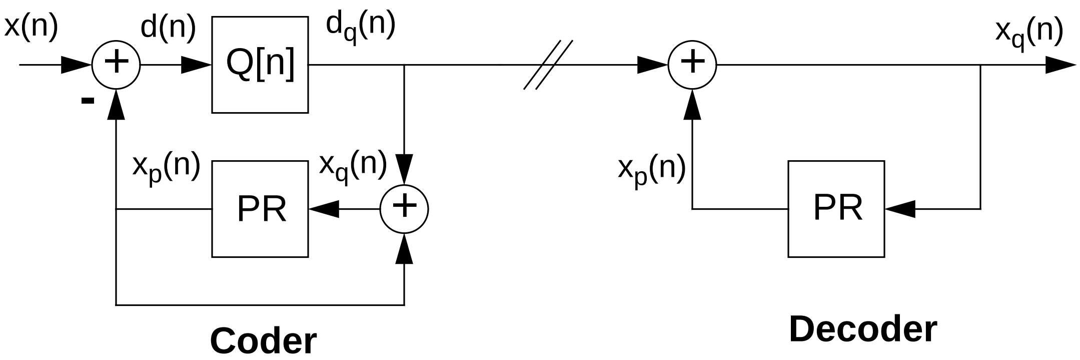
$$y = \begin{cases} \dfrac{Ax}{1+\log Ax} & \text{for } 0 \le |x| \le \dfrac{1}{A} \\ \dfrac{1+\log(Ax)}{1+\log A} & \text{for } \dfrac{1}{A} \le |x| \le 1 \end{cases} \qquad \text{A-law}$$





- 8 bits μ-law equivalent to 12 bits linear
- 8000 samples/second x 8 bits/sample = 64,000 bits/sec (codec)

Oversampling via Differential Sampling



Consider differential quantization:

$$d(n) = x(n) - ax(n-1)$$

It can be shown that the optimal value of a is given by:

$$a = \frac{\gamma_{xx}(1)}{\gamma_{xx}(0)} = \frac{\gamma_{xx}(1)}{\sigma_x^2}$$

and

$$\sigma_d^2 = \sigma_x^2[1 - a^2]$$

A more general predictor is a linear predictor:

$$\hat{x}(n) = \sum_{k=1}^{p} a_k x(n-k)$$

For the first-order predictor above,

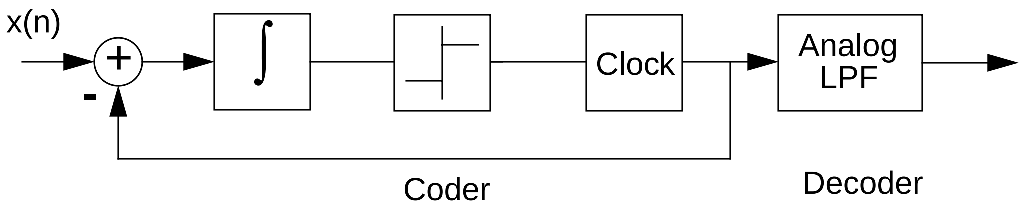
$$e(n) = d(n) - d_q(n) = x(n) - \hat{x}(n) - d_q(n) = x(n) - x_q(n)$$

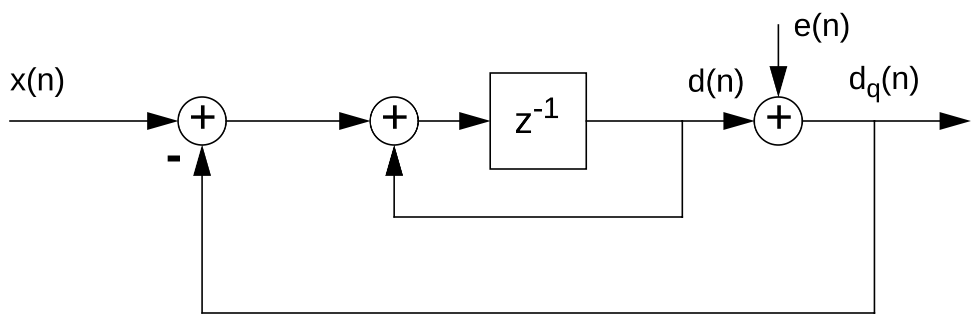
The error in the reconstructed signal is equal to the error in the difference signal.

One way to reduce the error is to raise the sample frequency so high that the signal is virtually constant within the difference interval (oversampling). A technique that exploits this property is delta modulation.

Sigma-Delta Modulation

Another way to reduce the error is to integrate the signal prior to conversion:





The integrator is modeled by:

$$H(z) = \frac{z^{-1}}{1 - z^{-1}}$$

The z-transform of the differential signal is:

$$\begin{aligned} D_q(z) &= \frac{H(z)}{1 + H(z)} X(z) + \frac{1}{1 + H(z)} E(z) \\ &= H_s(z) X(z) + H_n(z) E(z) \end{aligned}$$

Note that this gives a signal and noise component. The goal is to minimize the noise power by removing it from the signal band.

For the first order system above,

$$H_s(z) = z^{-1} \qquad H_n(z) = 1 - z^{-1}$$

The Performance of Sigma-Delta Converters

For a high sample frequency ($F_s \gg 2B$),

$$\sigma_n^2 = \int_{-B}^{B} \left|H_n(F)\right|^2 S_e(F)dF$$

where $S_e(F) = \sigma_e^2 / F_s$ is the power spectral density of the quantization noise.

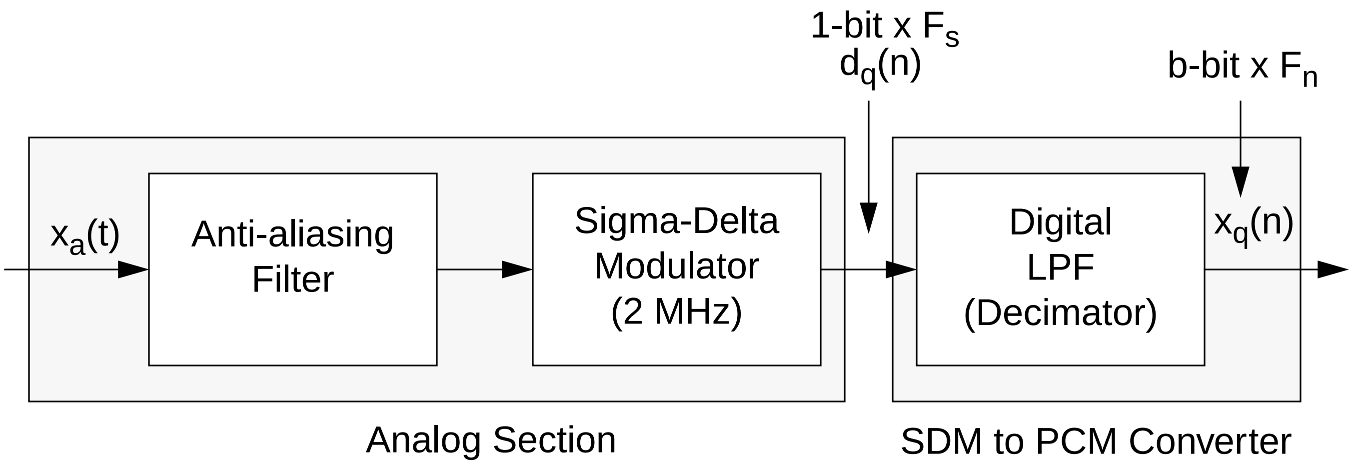
For the first order SDM, it can be shown:

$$\sigma_n^2 \approx \frac{1}{3}\pi^2\sigma_e^2\left(\frac{2B}{F_s}\right)^3$$

Note that the SNR goes down as the sample frequency increases.

Doubling the sample frequency drops the SNR by 9 dB.

An Oversampling A/D Converter



An Oversampling D/A Converter