LECTURE 33: SMOOTHING N-GRAM LANGUAGE MODELS

- Objectives:
  - Why do we need N-gram smoothing?
  - Deleted interpolation
  - Backoff language models
  - Discounting

This lecture combines material from the course textbook:

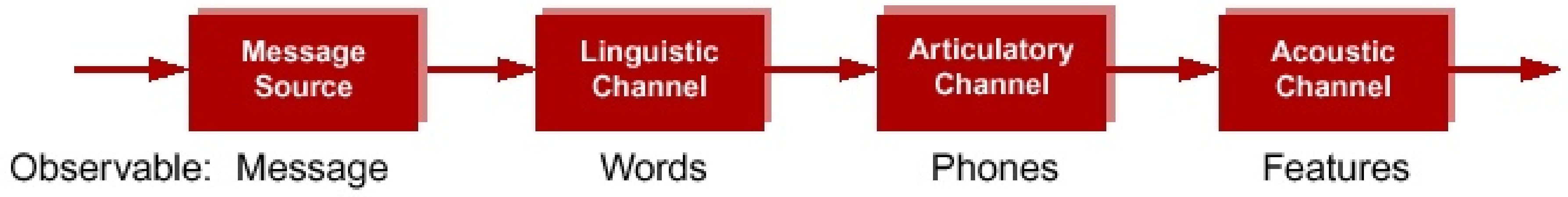
> X. Huang, A. Acero, and H.W. Hon, *Spoken Language Processing - A Guide to Theory, Algorithm, and System Development*, Prentice Hall, Upper Saddle River, New Jersey, USA, ISBN: 0-13-022616-5, 2001.

and from this source:

> F. Jelinek, *Statistical Methods for Speech Recognition*, MIT Press, Boston, Massachusetts, USA, ISBN: 0-262-10066-5, 1998.

A NOISY COMMUNICATION CHANNEL MODEL OF SPEECH RECOGNITION

A noisy communication theory model for speech production and perception:



Bayesian formulation for speech recognition:

$$P(W|A) = P(A|W)P(W)/P(A)$$

Objective: minimize the word error rate by maximizing $P(W|A)$

Approach: maximize $P(A|W)$ (training)

Components:

- $P(A|W)$: acoustic model (hidden Markov models, mixture of Gaussians)
- $P(W)$: language model (statistical, N-grams, finite state networks)
- $P(A)$: acoustics (ignore during maximization)

The language model typically predicts a small set of next words based on knowledge of a finite number of previous words (N-grams) — leads to search space reduction.

THE CHOMSKY HIERARCHY

We can categorize language models by their generative capacity:

Type of Grammar	Constraints	Automata
Phrase Structure	A -> B	Turing Machine (Unrestricted)
Context Sensitive	aAb -> aBb	Linear Bounded Automata (N-grams, Unification)
Context Free	A -> B Constraint: A is a non-terminal. Equivalent to: A -> w A -> BC where "w" is a terminal; B,C are non-terminals (Chomsky normal form)	Push down automata (JSGF, RTN, Chart Parsing)
Regular	A -> w A -> wB (Subset of CFG)	Finite-state automata (Network decoding)

- CFGs offer a good compromise between parsing efficiency and representational power.
- CFGs provide a natural bridge between speech recognition and natural language processing.

WHY IS SMOOTHING SO IMPORTANT?

- A key problem in N-gram modeling is the inherent data sparseness.
- For example, in several million words of English text, more than 50% of the trigrams occur only once; 80% of the trigrams occur less than five times (see SWB data also).
- Higher order N-gram models tend to be domain or application specific. Smoothing provides a way of generating generalized language models.
- If an N-gram is never observed in the training data, can it occur in the evaluation data set?
- Solution: **Smoothing** is the process of flattening a probability distribution implied by a language model so that all reasonable word sequences can occur with some probability. This often involves broadening the distribution by redistributing weight from high probability regions to zero probability regions.

SMOOTHING IS AN INTUITIVELY SIMPLE CONCEPT

- **Simple smoothing**: pretend each bigram occurs once more than it actually does in the training data set

$$P(w_i|w_{i-1}) = \frac{1 + C(w_{i-1}, w_i)}{\sum_{w_i}(1 + C(w_{i-1}, w_i))} = \frac{1 + C(w_{i-1}, w_i)}{V + \sum_{w_i} C(w_{i-1}, w_i)}$$

- Note that the probability density function must be balanced to that it still sums to one.

THE BACKOFF MODEL: A FLEXIBLE TRADE-OFF BETWEEN ACCURACY AND COMPLEXITY

- **Backoff smoothing**: Approximate the probability of an unobserved N-gram using more frequently occuring lower order N-grams

$$P_{smooth}(w_i | w_{i-n+1} \ldots w_{i-1}) =$$

$$\begin{cases} \alpha(w_i | w_{i-n+1} \ldots w_{i-1}) & C(w_{i-n+1} \ldots w_i) > 0 \\ \gamma(w_{i-n+1} \ldots w_{i-1}) P_{smooth}(w_i | w_{i-n+2} \ldots w_{i-1}) & C(w_{i-n+1} \ldots w_i) = 0 \end{cases}$$

- If an N-gram count is zero, we approximate its probability using a lower order N-gram.
- The scaling factor is chosen to make the conditional distribution sum to one.
- Extremely popular for N-gram modeling in speech recognition because you can control complexity as well as generalization.

DELETED INTERPOLATION SMOOTHING

- We can linearly interpolate a bigram and a unigram model as follows:

$$P_I(w_i \mid w_{i-1}) = \lambda P(w_i \mid w_{i-1}) + (1-\lambda)P(w_i)$$

- We can generalize this to interpolating an N-gram model using and (N-1)-gram model:

$$P_I(w_i \mid w_{i-n+1} \ldots w_{i-1}) = \lambda_{w_i \mid w_{i-n+1} \ldots w_{i-1}} P(w_i \mid w_{i-n+1} \ldots w_{i-1}) + (1 - \lambda_{w_i \mid w_{i-n+1} \ldots w_{i-1}}) P(w_i \mid w_{i-n+2} \ldots w_{i-1})$$

  Note that this leads to a recursive procedure if the lower order N-gram probability also doesn't exist. If necessary, everything can be estimated in terms of a unigram model.

- A scaling factor is used to make sure that the conditional distribution will sum to one.

- An N-gram specific weight is used. In practice, this would lead to far too many parameters to estimate. Hence, we need to cluster such weights (by word class perhaps), or in the extreme, use a single weight.

- The optimal value of the interpolation weight can be found using Baum's reestimation algorithm. However, Bahl *et al* suggest a simpler procedure that produces a comparable result. We demonstrate the procedure here for the case of a bigram laanguage model:

  1. Divide the total training data into kept and held-out data sets.

  2. Compute the relative frequency for the bigram and the unigram from the kept data.

  3. Compute the count for the bigram in the held-out data set.

  4. Find a weight by maximizing the likelihood:

$$\sum_{N(v) \in R} \sum_{w_2} C(v, w_2) \log(\lambda \times f(w_2 \mid w_1) + (1-\lambda) f(w_2 \mid w_1))$$

  This is equivalent to solving this equation:

$$\sum_{N(v) \in R} \sum_{w_2} N(v, w_2) \left[\lambda + \frac{f(w_2 \mid v)}{f(w_3) - f(w_2 \mid v)} \right]^{-1} = 0$$

The Good-Turing estimate states that for any N-gram, α, that occurs r times, we should reestimate this frequency of occurrence as:

$$r^* = (r+1)\frac{n_{r+1}}{n_r}$$

where n_r is the number of N-grams that occur exactly r times. This count can be convrted to a probability by dividing by the total number of N-gram tokens:

$$P(\alpha) = \frac{r^*}{N}$$

where

$$N = \sum_{r=0}^{\infty} n_r r = \sum_{r=0}^{\infty} n_r r^* = \sum_{r=0}^{\infty} (r+1)n_{r+1}.$$

The justification for this equation is as follows:

Suppose we have training data for N-grams $\alpha_1 \ldots \alpha_s$. Let $c(\alpha_i)$ denote the number of times the N-gram α_i occurs in the training data, and p_i be the true probability of α_i.

Estimating p_i by using its frequency of occurrence can be expanded as:

$$E(p_i|c(\alpha_i) = r) = \sum_{k=1}^{s} p(i = k|c(\alpha_i) = r)p_k$$

We are using the chance that a randomly selected N-gram, α_i, with count r, is in fact α_k. This can be rewritten into:

$$\begin{aligned} p(i = k|c(\alpha_i) = r) &= \frac{p(c(\alpha_k) = r)}{\sum_{l=1}^{s} p(c(\alpha_l) = r)} \\ &= \frac{\binom{N}{r}p_k^r(1-p_k)^{N-r}}{\sum_{l=1}^{s}\binom{N}{r}p_l^r(1-p_l)^{N-r}} \\ &= \frac{p_k^r(1-p_k)^{N-r}}{\sum_{l=1}^{s} p_l^r(1-p_l)^{N-r}} \end{aligned}$$

Substituting this into our expression for $E(p_i|c(\alpha_i) = r)$:

$$E(p_i|c(\alpha_i) = r) = \frac{\sum_{k=1}^{s}(p_k^r(1-p_k)^{N-r})p_k}{\sum_{l=1}^{s} p_l^r(1-p_l)^{N-r}}$$

Noting that every N-gram token counts as 1, we can express the expected value of n_r as:

$$E(n_r) = \sum_{i=1}^{s} p(c(\alpha_i) = r) = \sum_{i=1}^{s}\binom{N}{r}p_i^r(1-p_i)^{N-r}$$

We can show that:

$$\frac{r+1}{N+1}\frac{E_{N+1}(n_r+1)}{E_N(n_r)} = E(p_i|c(\alpha_i) = r)$$

We can make the approximation that $n_r \approx E_N(n_r)$ and

$$n_{r+1} \approx \frac{N}{N+1}E_{N+1}(n_{r+1})$$

Combining these results:

$$\begin{aligned} P(\alpha_i) &= \frac{r^*}{N} \\ r^* &= NP(\alpha_i) \\ &= NE(p_i|c(\alpha_i) = r) \\ &= N\frac{r+1}{N+1}\frac{E_{N+1}(n_r+1)}{E_N(n_r)} \\ &\approx (r+1)\frac{n_{r+1}}{n_r} \end{aligned}$$

Note that we must pre-smooth the distribution so $n_r > 0$.

KATZ SMOOTHING BASED ON GOOD-TURING ESTIMATES

- **Katz smoothing** applies Good-Turing estimates to the problem of backoff language models.
- Katz smoothing uses a form of *discounting* in which the amount of discounting is proportional to that predicted by the Good-Turing estimate.
- The total number of counts discounted in the global distribution is equal to the total number of counts that should be assigned to N-grams with zero counts according to the Good-Turing estimate (preserving the unit area constraint for the pdf).
- Katz Smoothing:

$$P_{Katz}(w_i|w_{i-1}) = \begin{cases} C(w_{i-1}w_i)/C(w_{i-1}) & r > k \\ d_r C(w_{i-1}w_i)/C(w_{i-1}) & k \geq r > 0 \\ \alpha(w_{i-1})P(w_i) & r = 0 \end{cases}$$

where $d_r = \dfrac{\dfrac{r^*}{r} - \dfrac{(k+1)n_{k+1}}{n_1}}{1 - \dfrac{(k+1)n_{k+1}}{n_1}}$ and $\alpha(w_{i-1}) = \dfrac{1 - \sum_{w_i : r > 0} P_{Katz}(w_i|w_{i-1})}{1 - \sum_{w_i : r > 0} P_{Katz}(w_i)}$.

KNESER-NEY BIGRAM SMOOTHING

- **Absolute discounting** involves subtracting a fixed discount, D, from each nonzero count, an redistributing this probability mass to N-grams with zero counts.
- We implement absolute discounting using an interpolated model:

$$P_{abs}(w_i | w_{i-n+1} \dots w_{i-1}) = \frac{max\{C(w_{i-n+1} \dots w_i) - D, 0\}}{\sum_{w_i} C(w_{i-n+1} \dots w_i)} + (1 - \lambda_{w_{i-n+1} \dots w_{i-1}}) P_{abs}(w_i | w_{i-n+2} \dots w_{i-1})$$

- **Kneser-Ney smoothing** combines notions of discounting with a backoff model. Here is an algorithm for bigram smoothing:

$$P_{KN}(w_i | w_{i-1}) = \begin{cases} \dfrac{max\{C(w_{i-1} w_i) - D, 0\}}{C(w_{i-1})} & C(w_{i-1} w_i) > 0 \\ \alpha(w_{i-1}) P_{KN}(w_i) & \text{otherwise} \end{cases}$$

where

$$P_{KN}(w_i) = \frac{C(\bullet w_i)}{\sum_{w_i} C(\bullet w_i)}$$

and $C(\bullet w_i)$ is the number of unique words preceding w_i.

$\alpha(w_{i-1})$ is chosen to make the distribution sum to 1:

$$\alpha(w_{i-1}) = \frac{1 - \sum_{w_i : C(w_{i-1} w_i) > 0} \dfrac{max\{C(w_{i-1} w_i) - D, 0\}}{C(w_{i-1})}}{1 - \sum_{w_i : C(w_{i-1} w_i) > 0} P_{KN}(w_i)}$$

- Knesser-Ney smoothing constructs a lower order distribution that is consistent with the smoothed higher order distribution.

CLASS N-GRAMS

- Recall we previously discussed defining equivalence classes for words that exhibit similar semantic and grammatical behavior.
- Class based language models have been shown to be effective for reducing memory requirements for real-time speech applications, and supporting rapid adaption of language models.
- A word probability can be conditioned on the previous N-1 word classes:

$$P(w_i | c_{i-n+1} \dots c_i) = P(w_i | c_i) P(c_i | c_{i-n+1} \dots c_i)$$

- We can express the probability of a word sequence in terms of class N-grams:

$$P(W) = \sum_{c_1 \dots c_n} \prod_i P(w_i | c_i) P(c_i | c_{i-n+1} \dots c_i)$$

- If the classes are non-overlapping:

$$P(W) = \prod_i P(w_i | c_i) P(c_i | c_{i-n+1} \dots c_i)$$

- If we consider the case of a bigram language model, we can derive a simple estimate for a bigram probability in terms of word and class counts:

$$\begin{aligned} P(w_i | w_{i-1}) \approx P(w_i | c_{i-1}) &= P(w_i | c_i) P(c_i | c_{i-1}) \\ &= \frac{C(w_i c_i)}{C(c_i)} \frac{C(c_{i-1} c_i)}{C(c_{i-1})} \\ &= \frac{C(w_i)}{C(c_i)} \frac{C(c_{i-1} c_i)}{C(c_{i-1})} \end{aligned}$$

- Class N-grams have not provided significant improvements in performance, but have provided a simple means of integrating linguistic knowledge and data-driven statistical knowledge.