

Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page (double-sided) of notes.
2. Please show ALL work. Answers with no supporting explanations or work will be given no credit.
3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

**Problem No. 1: Channel and Source Coding**

(a) For a binary symmetric channel, show that  $I(X;Y) \leq 1 - H(p)$ .

(b) Suppose the letters  $\{a,b,c,d\}$  are transmitted over this channel, and these letters have a prior distribution of  $p(x) = \{1/4, 1/4, 1/8, 3/8\}$ . Discuss the best way to send this data over the channel such that you minimize the error rate and you minimize the number of bits transmitted.

- (c) Suppose we add a second BSC with the same properties as the first. Derive an expression for the capacity, state whether the capacity increases or decreases, and explain why.

**Problem No. 2: Continuous Random Variables**

- (a) Prove the scaling theorem for the entropy of a continuous random variable.
- (b) Derive an expression for the capacity of a power-limited Gaussian channel (hint: compute the mutual information in terms of the entropies of the signal and noise, and apply bounds for these entropies).

- (c) Explain the significance of this result on three types of problems: compression, system identification, and maximum entropy spectral estimation.

(d) Explain Burg's Maximum Entropy Theorem.

**Problem No. 3: Statistics**

Consider a six-sided die containing the numbers  $\{1,2,3,4,5,6\}$ . You roll this die ten times and generate the sequence  $\{1,2,3,4,5,6,2,4,6\}$ .

(a) Describe the type class for this event.

(b) Bound the size of the type class.

- (c) Discuss the different ways to estimate the probability of the event above using concepts developed in this course. Be as precise as possible. Do not assume this is a fair die.