Exam #1 Rework

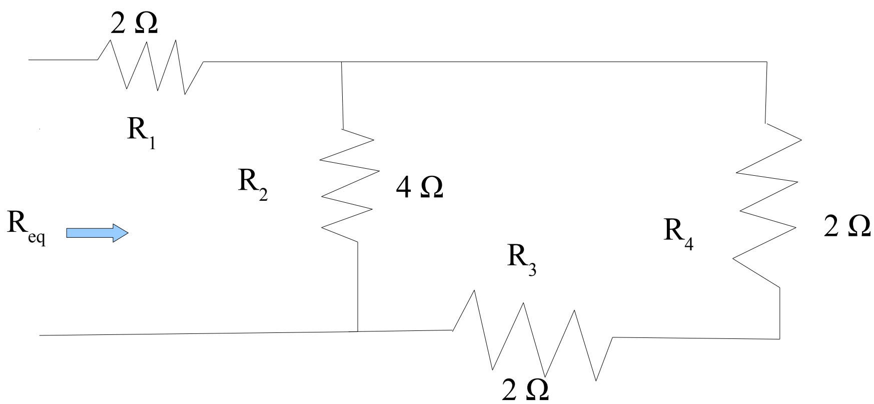
by Zack Smith

September 30th, 2012

ECE 2312: Electrical Engineering Science I

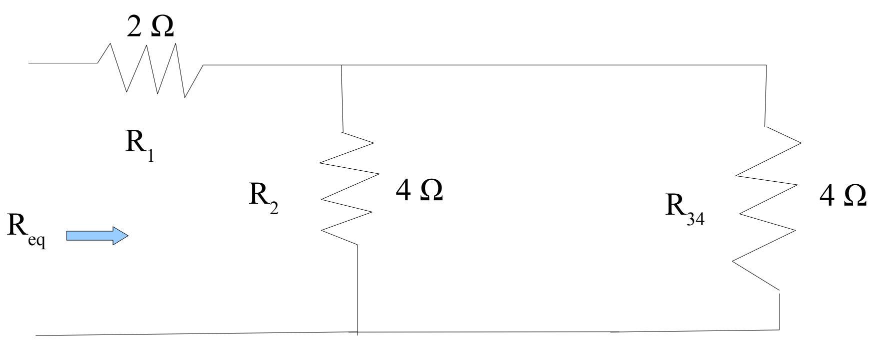
Problem #1a:

Compute the equivalent resistance for the network shown below:



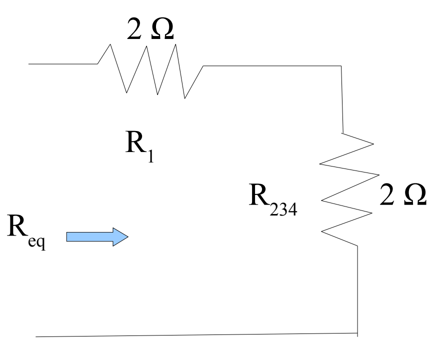
Firstly, we know R_3 and R_4 are in series, giving us a new equivalent resistor:

$R_3 + R_4 = 2\Omega + 2\Omega = 4\Omega$



Next, we know that R_{34} and R_2 are in parallel. This produces the equation for their equivalent resistance :

$(R_{34} * R_2)/(R_{34} + R_2) = (4\Omega * 4\Omega)/(4\Omega + 4\Omega) = 16\Omega/8\Omega = 2\,\Omega$

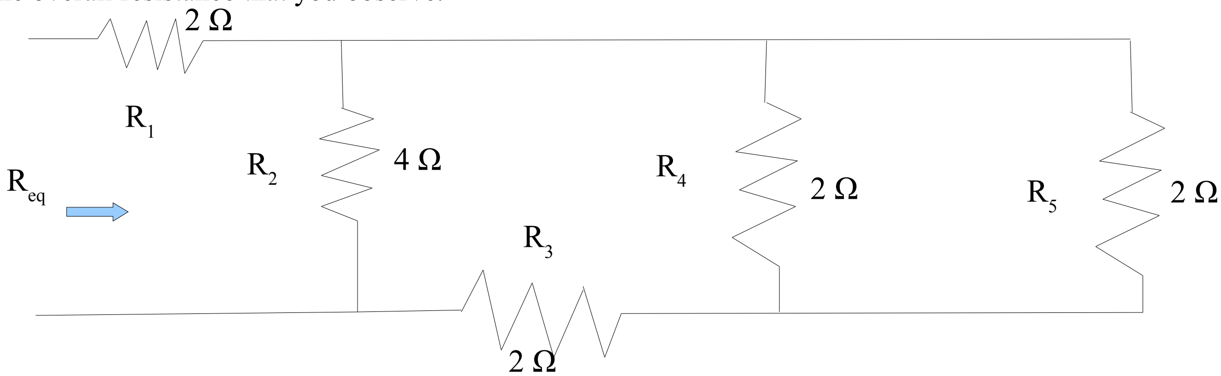


This new circuit gives us our final equation: R_1 and R_{234} are in series, so : $R_1 + R_{234} = 2\Omega + 2\Omega = 4\Omega$

This means R_{eq} equals 4Ω.

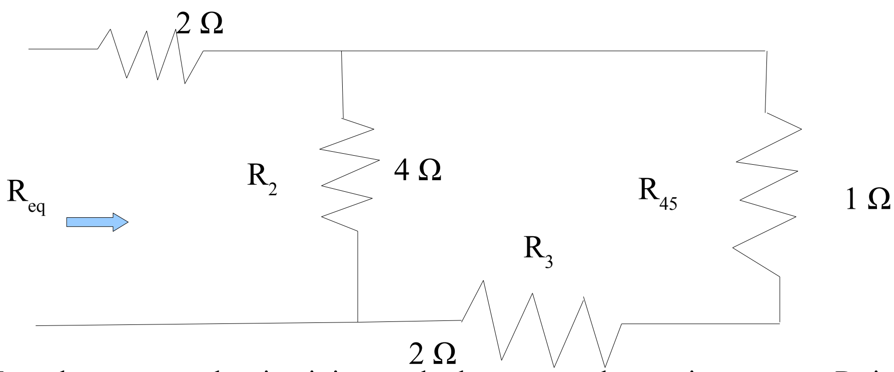
Problem #1b:

A 2Ω resistor is placed as a load on this network. Computer the equivalent resistance and explain any changes in the overall resistance that you observe.



In this case, R_5 and R_4 are in parallel. We can find their equivalent resistor with this equation:

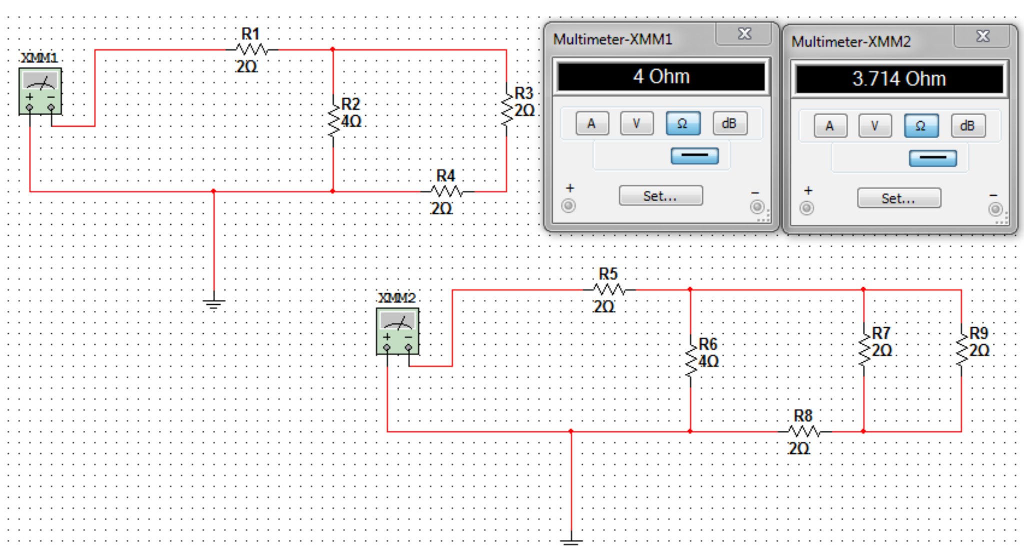
$$(R_5 * R_4)/(R_5+R_4)=(2\Omega*2\Omega)/(2\Omega+2\Omega)=4\Omega/4\Omega=1\Omega$$



From here on out, the circuit is exactly the same as the previous, except R_4 is now 1Ω instead of 2Ω. We can therefore use the same equations, substituting the new value, and obtain the new R_{eq}.

$$R_3+R_{45}=2\Omega+1\Omega=3\Omega$$

$$(R_{345}*R_2)/(R_{345}+R_2)=(4\Omega*3\Omega)/(3\Omega+4\Omega)=12\Omega/7\Omega$$

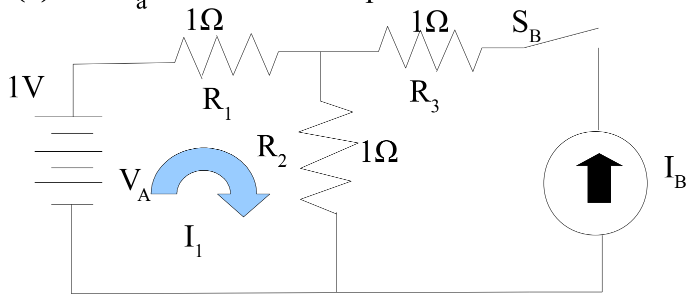


$$R_1+R_{2345}=2\Omega+12/7\Omega=14/7\Omega+12/7\Omega=26/7\Omega=3.714\Omega$$

Thus, the new R_{eq} = 26/7Ω. Our multisim circuit verifies these results:

Problem #2a:

2(a). Set V_a=1VDC and compute the current in the center resistor, I_c.



Since the switch, SB is open:

$$I_C=I_1$$

Therefore, we can do a KVL around the left inner loop:

$$R_1*I_1+R_2*I_C=1\text{V}$$

$$1\text{V}=2\text{I}_I$$

$$I_1=1/2\text{A}$$

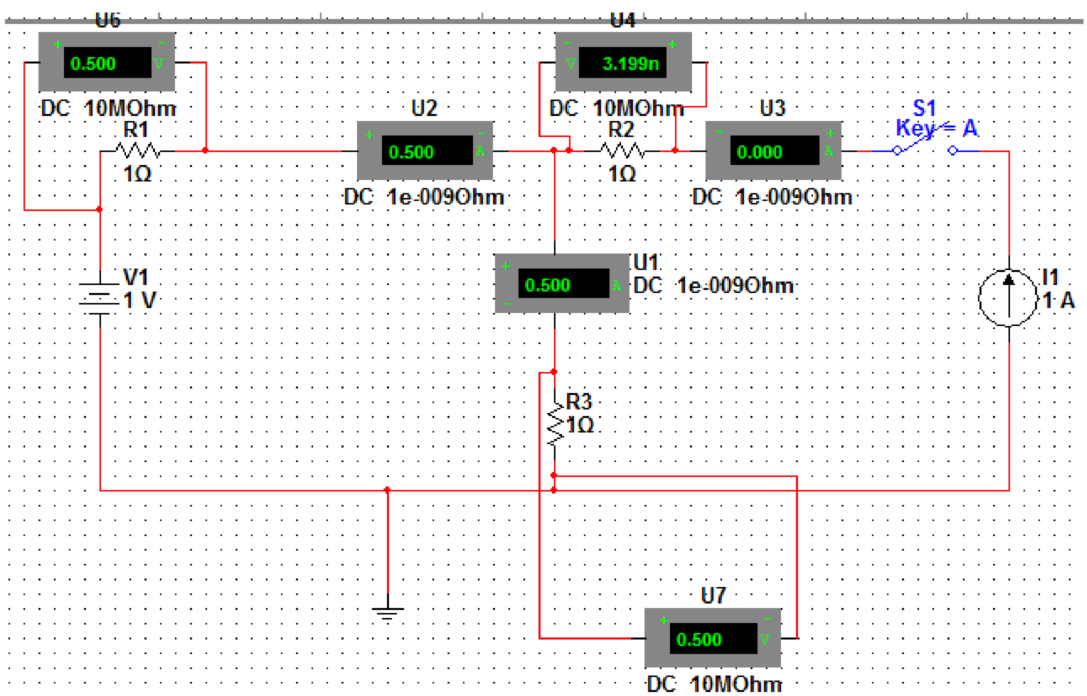
We therefore know that I_C = 1/2A

We can also analyze the voltages :

$$V=I*R$$

$$V_{R1}=1\Omega*I_1=.5\,V$$

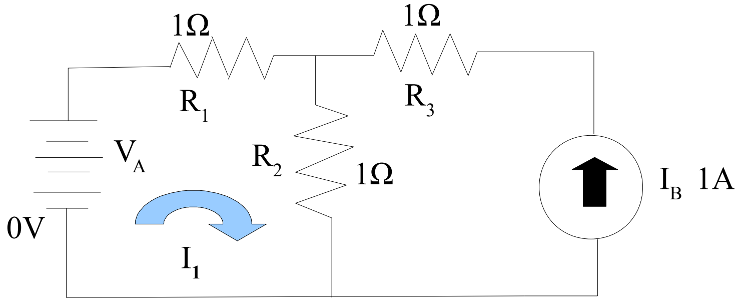
$$V_{R2}=1\Omega*I_C=.5\,V$$



Kirchhoff's voltage law checks out, there is 1V being output by the battery, and .5V is being used by each resistor. The sum of the voltage drops is therefore zero. Multisim confirms this:

Problem #2b:

Set V_a = 0 VDC, close the switch S_b, and set I_b=1A. Compute the current in the resistor, I_c.



Despite V_A becoming 0, there is still a current, I_1, running through the circuit along with I_B. You therefore cannot discount it, like we did with I_B in the previous part.

$$I_B + I_1 = I_C$$

We will use KVL:

$$I_1 * R_1 - V_A + I_C = 0 \qquad I_1 + I_B + I_1 = 0$$

$$2I_1 = -I_B \qquad I_1 = -1/2\text{A}$$

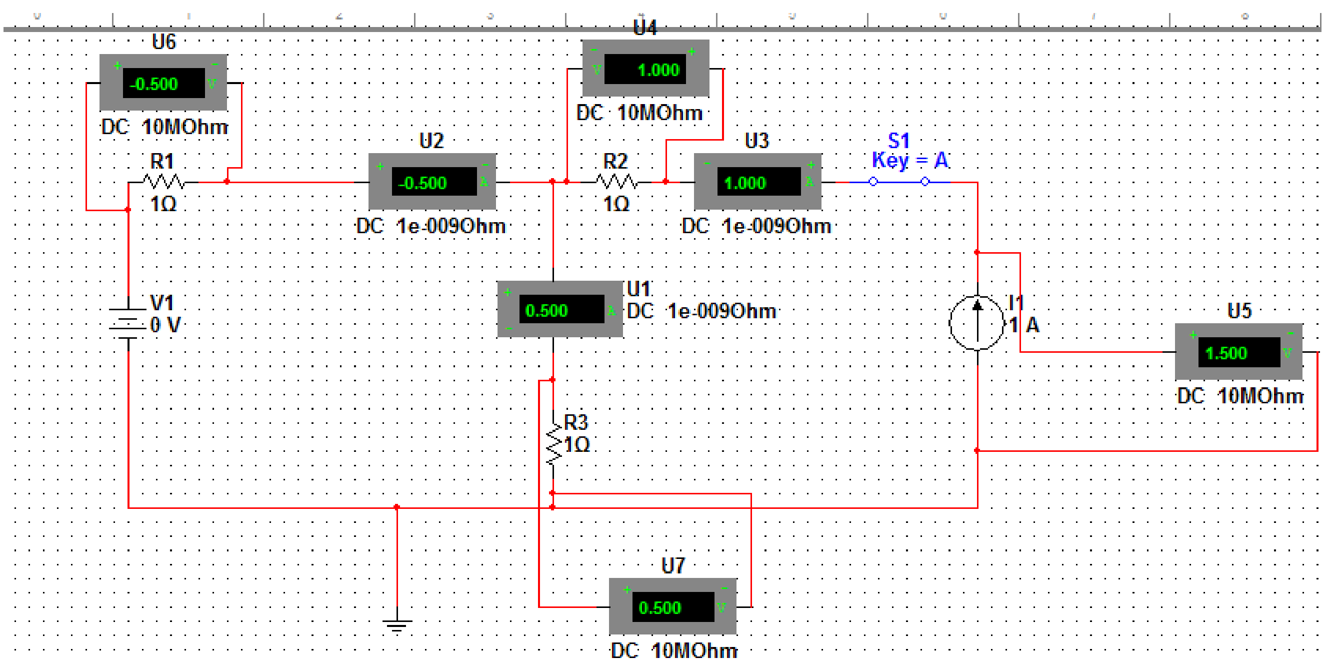
$$I_B + I_1 = I_C \qquad 1\text{A} - 1/2\text{A} = I_C$$

Thus, I_C = 1/2A, yet again.

We can find the voltages across the resistors :

$$V_{R3} = I_B * 1\Omega = 1\text{V} \qquad V_{R1} = I_1 * R_1 = -1/2\text{V} \qquad V_{R2} = 1\Omega * I_C = 1/2\text{V}$$

Summing up the voltages across either loop will result in the Voltage supplied by I_B, which is 1.5V.



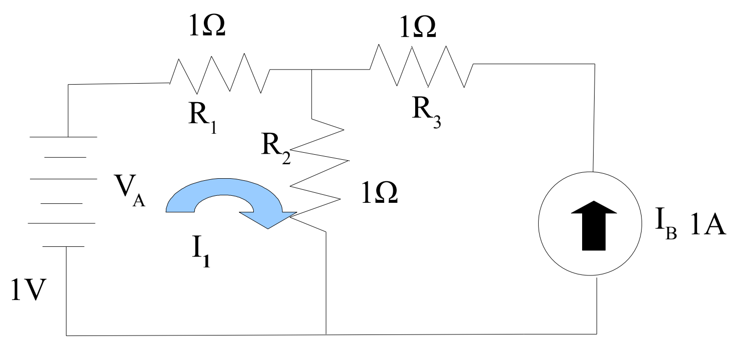
KCL also holds at the node connecting all the resistors:

$$1\text{A}(\textit{flowing into the node}) = .5\,A + .5\,A(\textit{flowing out})$$

Multisim confirms all of this:

Problem #2c:

2(c).Set V_a=1VDC, close the switch S_b, and set I_b=1A. Compute the current in the resistor, I_c.



We can now do the KVL to get I_C when both the sources are on:

$$-V_a+I_1+I_c$$

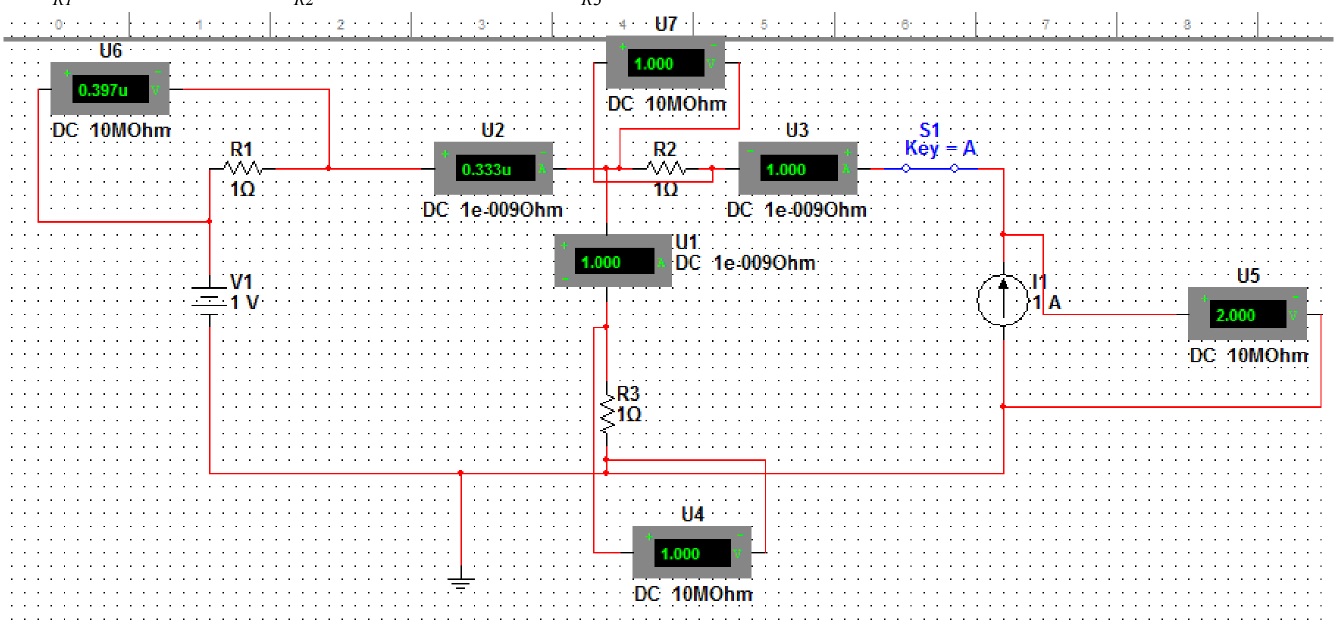
$$1=2I_1+1$$

$$I_1=0$$

$$I_C=I_1+I_B=1\text{A}$$

We can also find the voltages across the resistors:

$$V_{R1}=1\Omega*0=0 \qquad V_{R2}=1\Omega*1\text{A}=1\text{V} \qquad V_{R3}=1\Omega*1\text{A}=1\text{V}$$



Summing the voltages, we would get that the voltage across the current source would be 2V. Multisim confirms this:

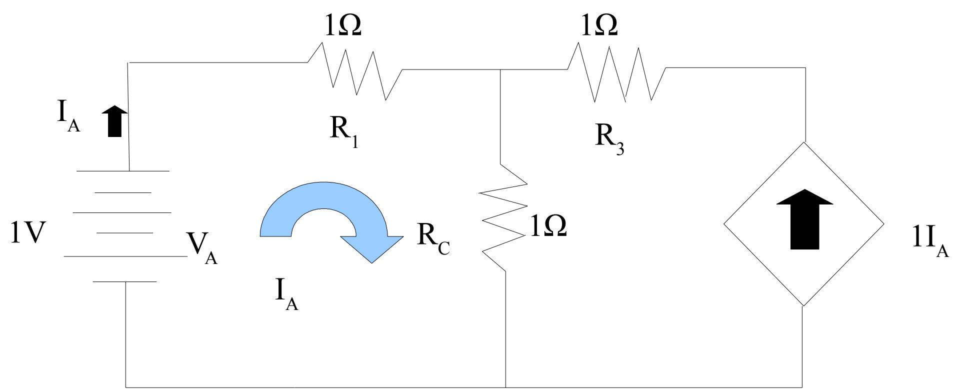
Problem #2d:

2(d). Does the result of (c) equal the sum of 2(a) and 2(b)? Explain any differences that you observe.

Yes, my result for part c was 1A, and the sum of 2(a) and 2(b) is 1A as well. Why this is the case is simple. Individually, both sources supply 1/2A to the resistor, both in the same direction. It therefore stands to reason that when they're both on, together, they will supply 1/2A + 1/2A to the resistor, or 1A. This may not be true for dependent sources or non-linear elements, but for this circuit, which has neither, it applies perfectly.

Problem #3a:

3(a). Compute the power dissipated in R_c.



This problem is perfectly suited for solving via loop analysis. We can say that the current flowing through the left loop is I_A, and the right loop is also I_A. We can write the left loop equation like this:

$$-V_A + I_A * 1\Omega + 1\Omega * (I_A + I_A)$$

Which simplifies to : $V_A = 3I_A$ $\quad I_A = 1/3\text{A}$

Now we can find the power running through R_C

$$P = I^2 R \quad I_{RC} = I_A + I_A = 2I_A = 2/3 \quad P_{R_C} = I_{R_C}^2 * R_{R_C} = (2/3)^2 * 1\Omega = 4/9\,\mathit{Watts}$$

We can also find the voltage across all of the circuit elements:

$$V_{R1} = I_A * R_1 = 1/3\text{A} * 1\Omega = 1/3\text{V} \quad V_{RC} = 2I_A * R_C = 2 * 1/3 * 1\Omega = 2/3\,V$$

$$V_{R3} = I_A * R_3 = 1/3\text{A} * 1\Omega = 1/3\text{V}$$

To find voltage across the current source, we must do another loop:

$$V_{CS} = I_A * R_3 + 2I_A * R_C = 1/3\text{A} * 1\Omega + 2/3\text{A} * 1\Omega = 1\text{V}$$

If we do any loop, we will find that the sum of the voltages is equal to zero. Furthermore, KCL at the node connecting all three resistors holds true: $I_A + I_A\,\mathit{Flowing\ into\ the\ node} = 2I_A\,\mathit{Flowing\ out}$

Problem #3b:

3(b). Compute the power delivered by the source, V_a.

The power supplied by the power source is simple:

$$P = V * I$$

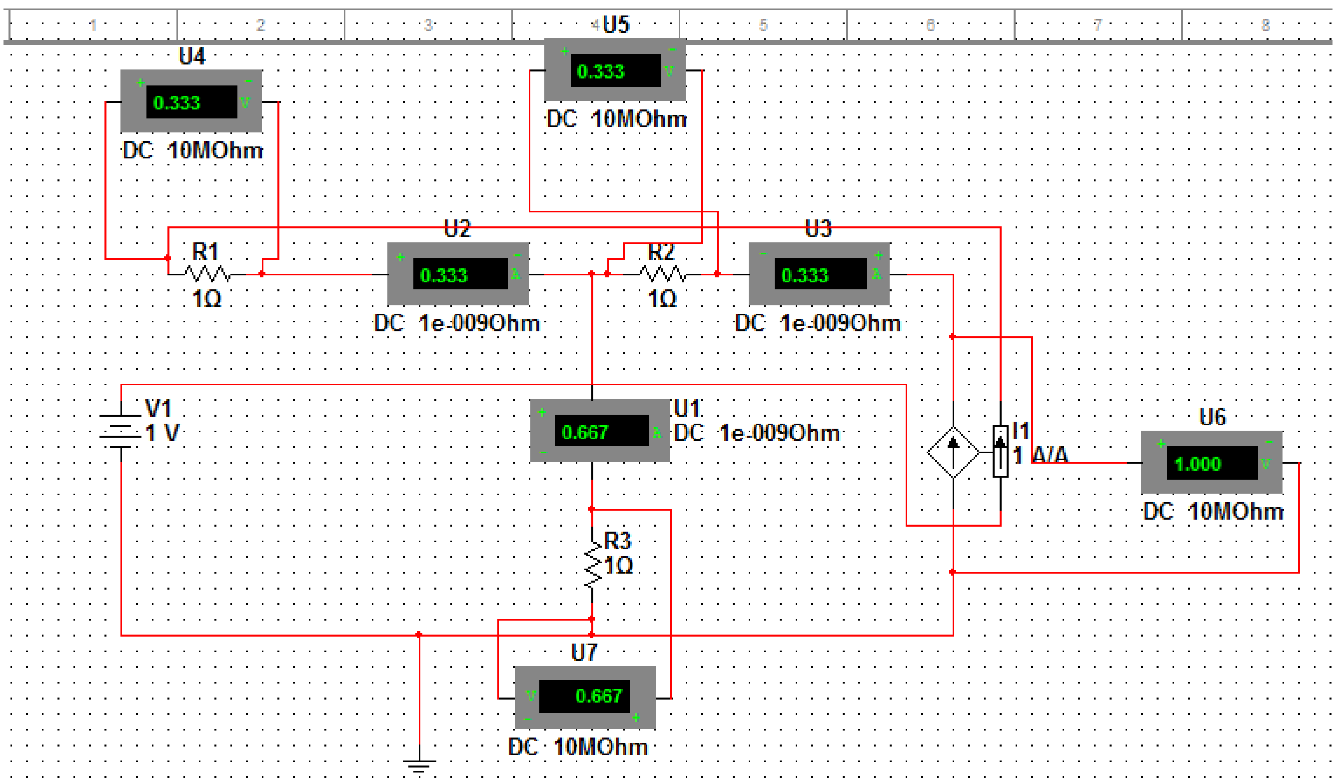
$$P_{V_A} = I_A * V_A = 1/3\text{A} * -1\text{V} = -1/3\,Watts$$

So the battery is supplying 1/3 Watts of power to the circuit. Likewise, the current source is also supplying 1/3 Watts of power to the circuit.
We should evaluate the power dissipated by all the resistors, because if it does not equal the power supplied by the sources, it means there is an error in the analysis.

$$P_{R_1} = V_{R_1} * I_{R_1} = 1/3\text{V} * 1/3\text{A} = 1/9\,Watts$$

$$P_{R_3} = V_{R_3} * I_{R_3} = 1/3\text{V} * 1/3\text{A} = 1/9\,Watts$$

$$P_{R_C} = V_{R_C} * I_{R_C} = 2/3\text{V} * 2/3\text{A} = 4/9\,Watts$$



If we add up all the power dissipated by the resistors we get 6/9 Watts dissipated. If we sum up the power supplied by the sources, we get 6/9 Watts supplied. This confirms that our values for power are not incorrect. Multisim Confirms this:
Through multisim, our voltage, current, and power calculations are all validated.