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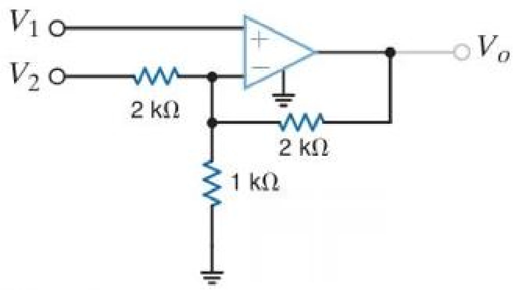
EE Science 1
11/1/2012

Reworked Exam

1. Derive an expression for V_O as a function of V_1 and V_2.



It is important to consider the fact that the ideal op amp model conditions are applied ($V_- = V_+$, and $i_- = i_+ = 0$)

If KVL is applied in the node located at inverting terminal the following equation can be derived:

$$\frac{-V_2 - V_-}{2K\Omega} + \frac{V_-}{1K\Omega} - \frac{-V_O - V_-}{2K\Omega} = 0$$

It is possible to solve for V_o' from the previous equation:

The fractions are separated in order to simplify the calculations:

$$\frac{-V_2}{2K\Omega} + \frac{2V_-}{2K\Omega} + \frac{V_-}{1K\Omega} - \frac{V_o}{2K\Omega} = 0$$

If the terms are combined and the term that contains V_o is placed in the right side of the equation, the equation can be written as:

$2V_- - V_2 = \frac{V_o}{2K\Omega}$ solving for V_o we obtain:

$2(2V_- - V_2) = V_o$, which yields

$4V_- - V_2 = V_o$ **(1)**

Since the answer must be in terms of V_1 and V_2, a second equation can be derived from the noninverting terminal in order to solve for V_- in terms of V_1 and V_2.

At this point we can write that:

$V_- = V_+ = V_1$ **(2)**

Therefore, if this equation is substituted in equation 1 we can say that

$$4V_1 - V_2 = V_o$$

The Multisim simulation for this circuit proves that the equation derived is accurate. If $V_1 = 2V$ and $V_2 = 1V$ the equation predicts that the output should be 7V. Figure 1 shows that this is accurate.

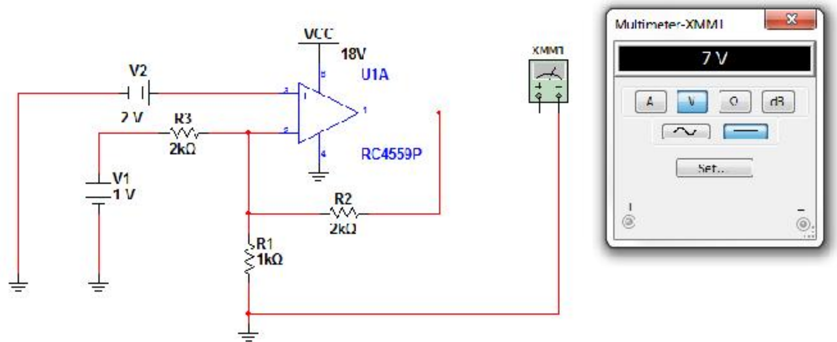
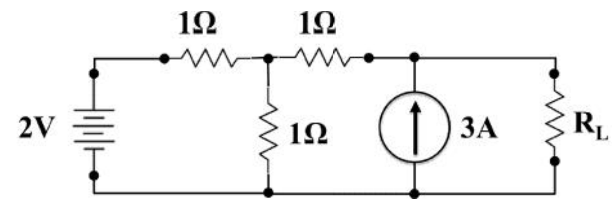


Figure 1.

2. (a) Using superposition, find the value of R_L that maximizes the power dissipated in R_L. Compute the portion of this power due to the voltage source only, and the portion due to the current source.



2.a. If the current source is opened, the circuit shown in Figure 2 is obtained:

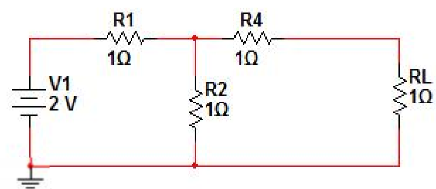


Figure 2.

In order to find the value of R_L that maximized the power dissipated by the load, it is necessary to find R_{TH}, because, according to the Maximum Power Transfer Theorem, the maximum power is achieved when R_L is equal to R_{TH}.

In this sense, the circuit was opened at R_L and the voltage of the open circuit V_{oc} was found through the following analysis:

Using KCL around the leftmost mesh:

-2V + 1ΩI + 1ΩI = 0 Solving for I,

2ΩI = 2V and therefore I = 1A.

This calculation can be corroborated in Multisim, as is shown in Figure 3.

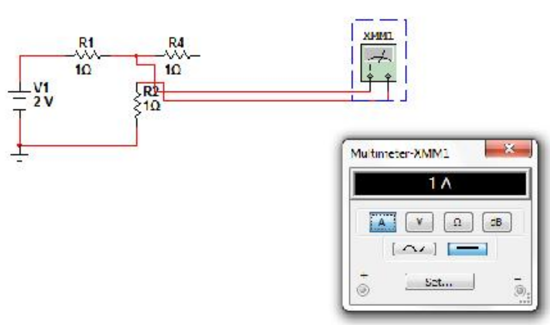


Figure 3.

Since there is not current flowing through Resistor 4, it can be easily determined that the V_{oc} is given by IR_2, which is:

(1A)(1Ω) = 1V

The veracity of this equation is equally proven by the Multisim simulation, which is shown in Figure 4.

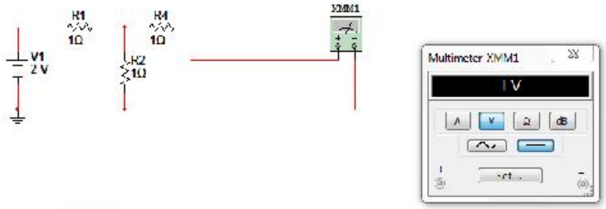


Figure 4.

It is possible to solve for R_{TH} by setting all the sources to zero and calculating the equivalent resistance.

Since there are two 1Ω resistances in parallel and another 1Ω resistance in series with the previous configuration, it can be said that R_{TH} is given by:

$R_{TH} =$ (1Ω||1Ω) + 1Ω, which is $R_{TH} = (\frac{1}{2} + 1)\Omega$ or $R_{TH} = 1.5\Omega$

Figure 5 shows how the result obtained for R_{TH} is corroborated in Multisim.

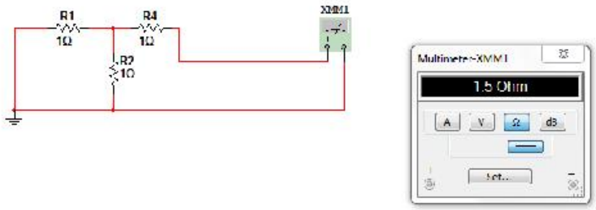


Figure 5.

In this sense the Thevenin circuit for the voltage source contribution is shown in Figure 6, and R_L is set to the value of R_{TH} in order to maximize the power dissipated by it.

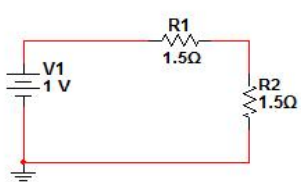


Figure 6.

Figure 7 shows the diagram of the circuit when the voltage source is shorted.

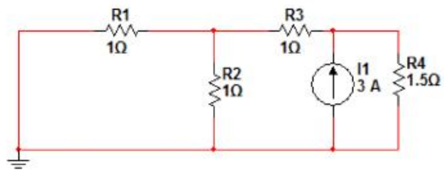


Figure 7.

This circuit can be redrawn as shown in Figure 8:

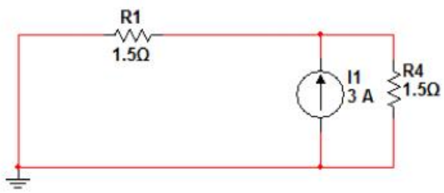


Figure 8.

The current going passing through R4 can be calculated through Current Division:

In this sense,

$I = 3(\frac{1.5}{1.5+1.5})$ which yields I = 1.5A (This can be seen in Figure 9)

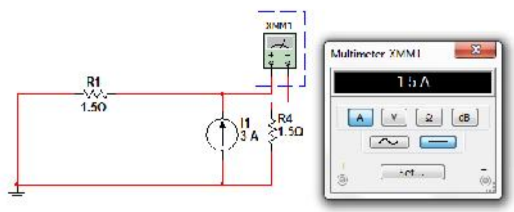


Figure 9.

In this sense, the power dissipated by the load is given by $P = (I^2R_L)$ or $P = (1.5^2(1.5))$ P=3.375W.

The power was also calculated for the Voltage source contribution. The diagram of this circuit is presented in Figure 10.

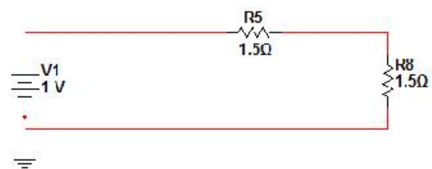


Figure 10.

The current through the resistor can be simply calculated through Ohm's law.

$I = \frac{V}{R}$ or

$I = \frac{1V}{(1.5+1.5)A}$ which is I = $\frac{1}{3}A$

Figure 11 shows the Multisim verification of the current calculated.

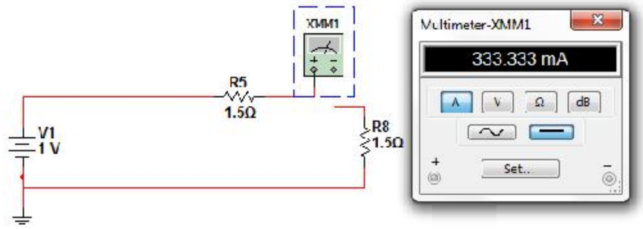


Figure 11.

In this sense, the power going through the resistor is $P = (1/3)^2(1.5))$, which is P = 1/6W.

The parameter sweep shown in Figure 12 demonstrates that the value calculated for R_{TH} is accurate.

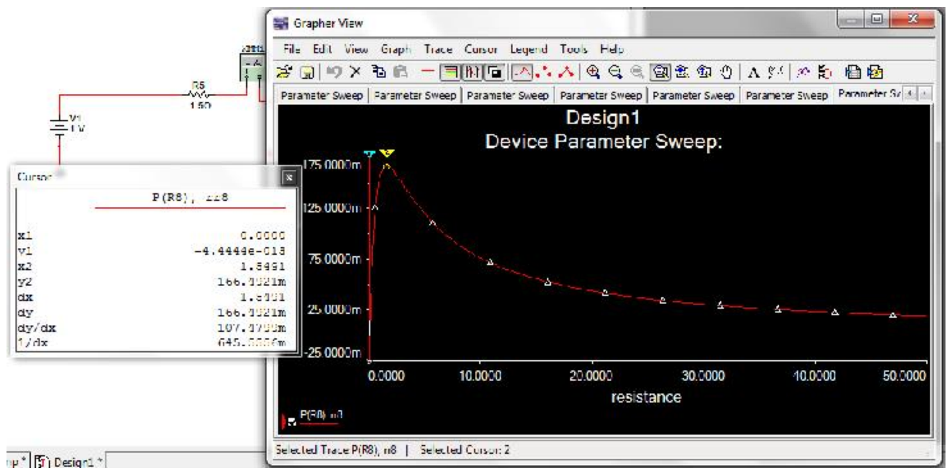


Figure 12.

2(b). Remove the current source by treating it as an open circuit. Find the value of RL that maximizes the power dissipated in RL. Explain why this value is the same, or different, than the answer to (a). Justify your conclusion whether it should be the same or different.

Figure 13. shows how the circuit looks like when the current source is opened.

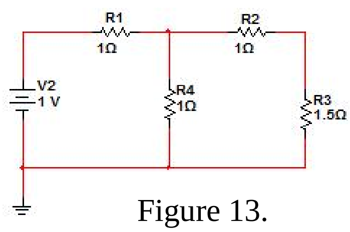


Figure 13.

In this case, the R_{TH} is found in the same way it was found before by the current contribution in the first part of the question:

Using KCL around the leftmost mesh:

-2V + 1ΩI + 1ΩI = 0 Solving for I,

2ΩI = 2V and therefore I = 1A.

(1A)(1Ω) = 1V

Since there are two 1Ω resistances in parallel and another 1Ω resistance in series with the previous configuration, it can be said that R_{TH} is given by:

$R_{TH} = (1\Omega || 1\Omega) + 1\Omega$, which is $R_{TH} = (\frac{1}{2} + 1)\Omega$ or $R_{TH} = 1.5\Omega$

If $R_L = R_{TH}$ and the current through R_L in the Thevenin circuit (shown in Figure 10) is calculated through Ohm's law it can be obtained that the current is 1/3A and therefore, the power would be P = 1/6W.

The power calculated in this question is the same as the power calculated for the contribution of the voltage source, for the fact that, in both cases, the current source was opened and the components in the resulting circuit remained the same.

3. Find the energy stored in the 2F capacitor and 3H inductor. (Hint: find all voltages and currents first.)

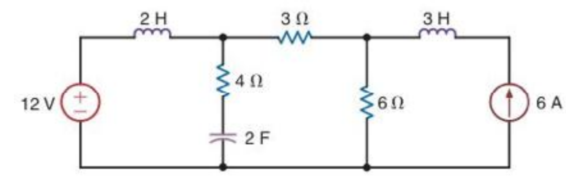


Figure 14 shows the diagram for the circuit at time infinity, when the capacitors act as open circuits and the inductors act as short circuits.

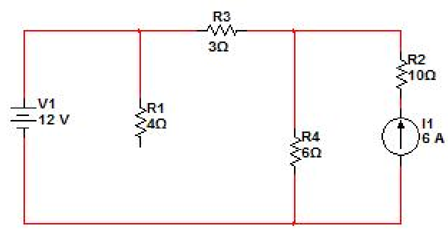


Figure 14.

The open circuit voltage can be calculated through KVL around the leftmost mesh:

Since there is not a current flowing through R1, the KVL equation for this loop is reduced to

-12V + V_{oc} = 0, therefore $V_{oc} = 12V$

This can be seen in the Multisim simulation presented in Figure 15.

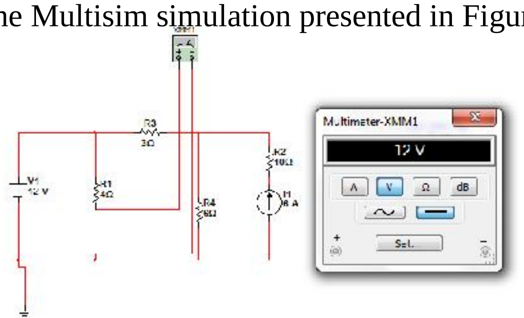


Figure 15.

The Short circuit current through the 3H inductor can also be easily calculated, because it is in the same loop as the current source.

Therefore,

$I_{sc} = 6A$

Figure 16 shows the Multisim simulation for this calculation.

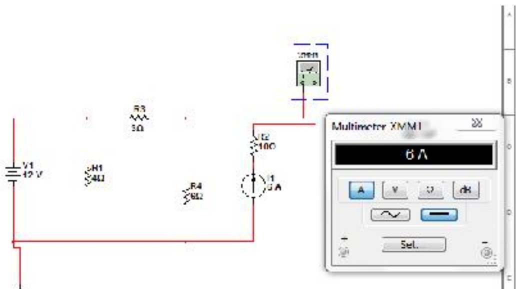


Figure 16.

In this sense, the energy stored in the Capacitor is given by

W = $\frac{1}{2}C(V^2)$ or W = $\frac{1}{2}2(12^2)$ = 144J

On the other hand, the energy stored by the 3H inductor is given by

W = $\frac{1}{2}L(I^2)$ or W = $\frac{1}{2}3(6^2)$ = 54J