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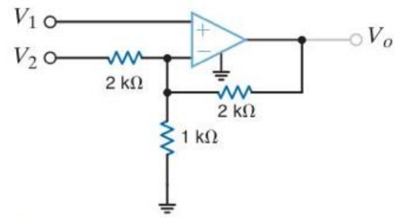
Document: Exam 2 Rework

1. Derive an expression for V_o as a function of V_1 and V_2.

EXPLANATION



In order to solve for the output voltage V_o we assume that the conditions imposed on this circuit are ideal settings for this operational amplifier. These conditions are as follows: the current entering and exiting the operational amplifier is zero and the voltage at the inverting terminal of the operational amplifier is equivalent to the voltage at the noninverting terminal of the operational amplifier.

The first step of solving this problem is to redraw the diagram. We redraw the diagram, substitute the values of the resistors with variables in our equations and label all other variables and nodes. Refer to Figure 1.

Now that we have done this, we establish all of the facts pertaining to the circuit. After assuming ideal operational amplifier conditions, we know the following:

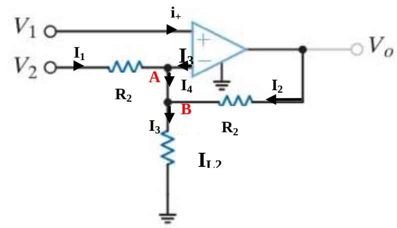


Figure 1

- $\mathbf{i_+ = i_- = 0}$
- $\mathbf{V_+ = V_1}$
- $\mathbf{V_+ = V_-}$

The third step is to find all of the currents within the circuit. We know i_+ and i_- are zero. We used Ohm's Law (***V=IR***) to calculate all of the other currents in the circuit. All other currents found through the use of the formula ***(*** $\boldsymbol{I = \frac{V}{R}}$ ***)*** are listed below**.**

- $\mathbf{i_+ = i_- = 0}$
- $I_1 = \frac{(V_2 - V_-)}{R}$
- $I_2 = \frac{(V_o - V_-)}{R}$
- $I_3 = \frac{(V_- - 0)}{R}$

The next step is to perform nodal analysis so that we can develop equations that can be solved for the output voltage V_o. By applying Kirchoff's Current Law to each node, we find the equations below.

KCL @ node A: $- I_1 - i_- + I_4 = 0$

KCL @ node B: $- I_4 - I_2 + I_3 = 0$

At node A, we find that I_4 is equivalent to I_1 due to the fact that i_- equals zero. At node B, we find that I_2 is equivalent to the difference between I_4 and I_3. Now we can substitute so that the node equations above are in terms of voltages.

We get the following:

$$\frac{-(V_2 - V^-)}{R_2} - \frac{(V_0 - V^-)}{R_2} + \frac{V^-}{R_1}$$

$$\frac{-(V_0 + V_1)}{R_2} = \frac{(V_2 - V_1)}{R_2} - \frac{V^-}{R_1}$$

$$-V_0 = V_2 - V_1 - \frac{V_1 R_2}{R_1} - V_1$$

$$V_0 = 2V_1 + \frac{V_1 R_2}{R_1} - V_2$$

$$V_0 = 2V_1 + 2V_1 - V_2$$

$$V_0 = 4V_1 - V_2$$

We find that the output voltage simplifies to: $V_0 = 4V_1 - V_2$.

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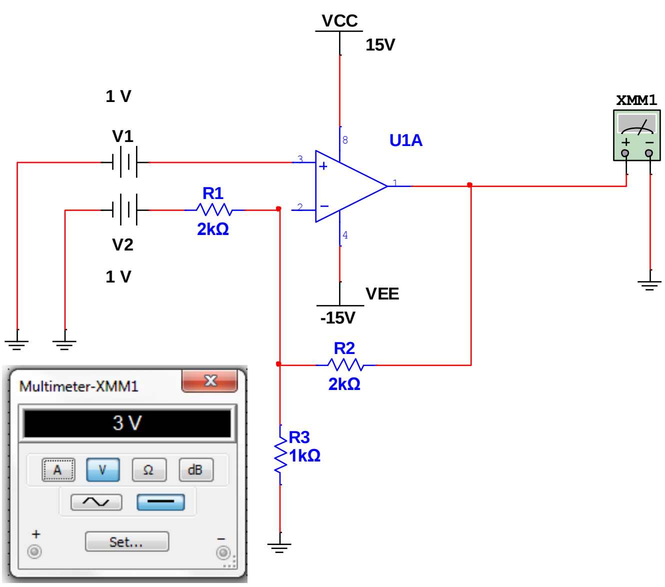
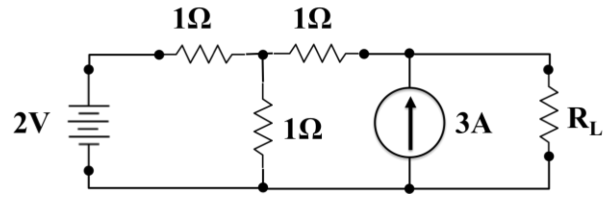


Figure 2

As you can see from Figure 2, when V_1 and V_2 are 1V, R_1 and R_2 are 2kΩ and R_3 is 1kΩ, the output voltage is 3V. This is consistent with our formula to calculate the output voltage:

$V_0 = 4(1)-1(1) = 3V$

2. (a) Using superposition, **f**ind the value of R_L that maximizes the power dissipated in R_L. Compute the portion of this power due to the voltage source only, and the portion due to the current source.



EXPLANATION

We will use superposition to find the value of the R_L

Step 1) Find the Thevenin Resistance

- Remove the Load
- Open the Current, Short the Voltage
- Redraw
- Reduce the circuit to find R_{TH}

First we remove the load in order to establish an equivalent circuit to the left of the load. After we remove the load, we short the voltage, open the current, redraw the circuit and reduce it to find R_{TH}. We reduced the circuit and found that the Thevenin Resistance is $\frac{3}{2}\Omega$. We used the formula: $RTH = \frac{(R1*R2)}{(R1+R2)} + \text{R3} = \frac{3}{2}\Omega$. The reason we know that R_L (the load resistance) must be equivalent to R_{TH} is due to the maximum power transfer theorem. Refer to Figure 3.

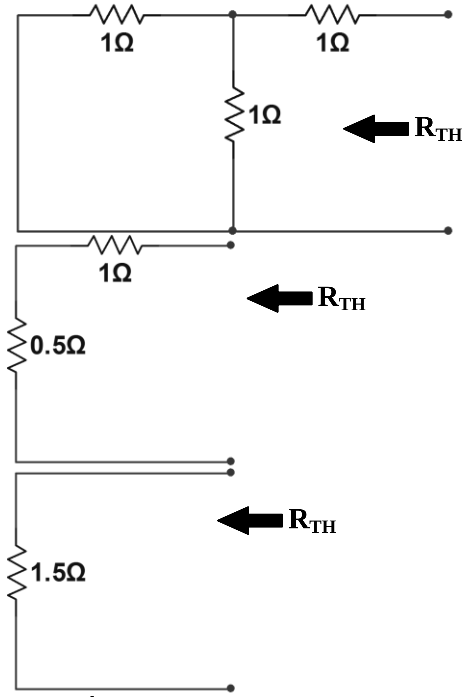


Figure 3

Step 2) Find the Power Contribution of the Voltage Source Only

- Rebuild equivalent circuit
- Connect R_L and R_{TH} in series

Now that we have a reduced circuit, we can find the power dissipated though the load resistor. We already know that the formula for power is as follows: $P = I^2 R_L$. By performing KVL on the loop, we can find that $I = \frac{2}{3}$ based on Ohm's Law. Now we can substitute to find the power dissipated in the load resistor to get power; $P = \frac{4}{9} * \frac{3}{2} = \frac{2}{3}$. Refer to Figure 4.

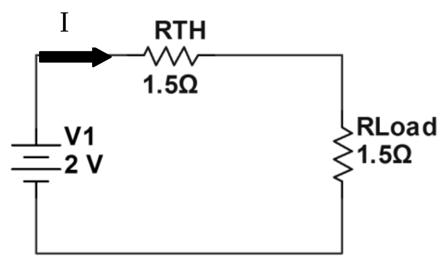


Figure 4

Step 3) Find the Power Contribution of the Current Source Only

- Rebuild equivalent circuit
- Connect R_L and R_{TH} in parallel

Now that we have a reduced circuit, we can find the power dissipated though the load resistor. We already know that the formula for power is as follows: $P = I^2 R_L$. By performing KVL on the loop, we can find that $I = \frac{3}{2}$ based on the theory of current division. Now we can substitute to find the power dissipated in the load resistor to get $P = \frac{9}{4} * \frac{3}{2} = \frac{27}{8}$. Refer to Figure 5.

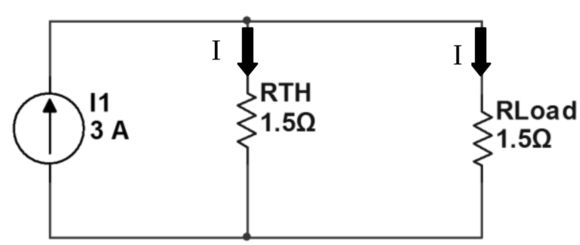


Figure 5

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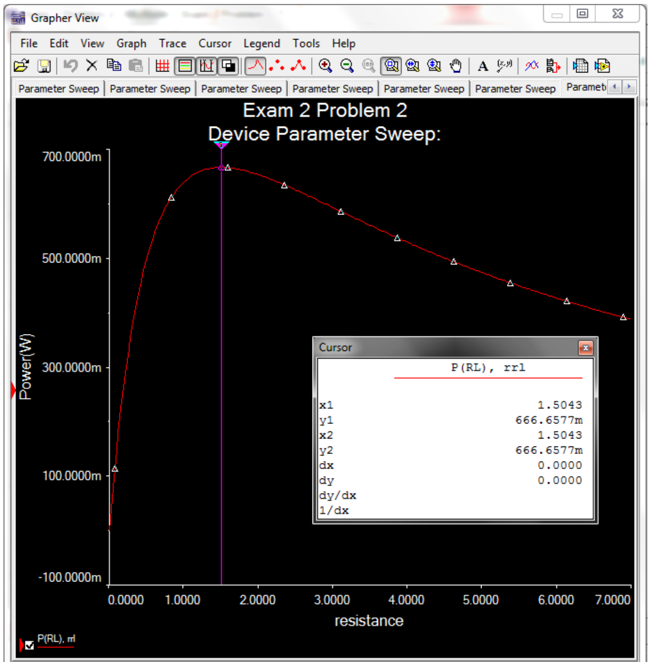

Figure 6

As you can see from Figure 6, the maximum power transferred across the load occurs when the load resistance is equivalent to the Thevenin Resistance. We used the parameter sweep to vary the load resistance in the circuit to find the optimal transfer of power. This result is consistent with the maximum power transfer theorem.

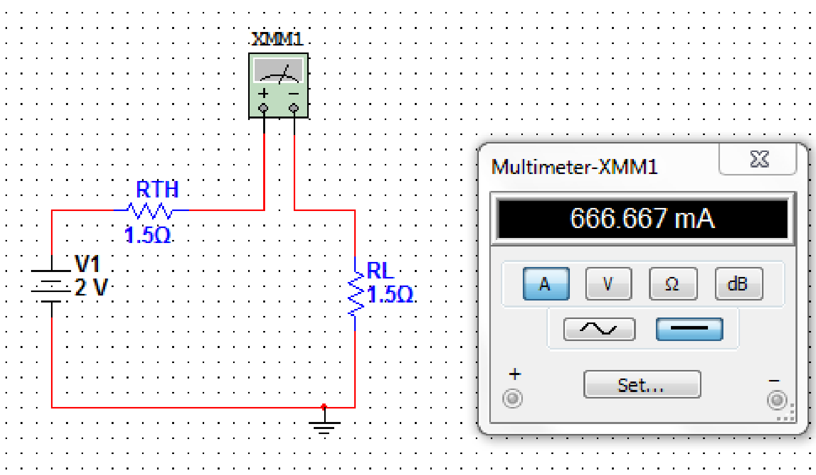

Figure 7

As you can see from Figure 7, our simulation matches the results that we obtained by using superposition. We obtain a current of $\frac{2}{3}$A when we analyze the voltage source's contribution to the power dissipated in R_L. Since the resistors are connected in series, current throughout the circuit is the same I = $\frac{2}{3}$A.

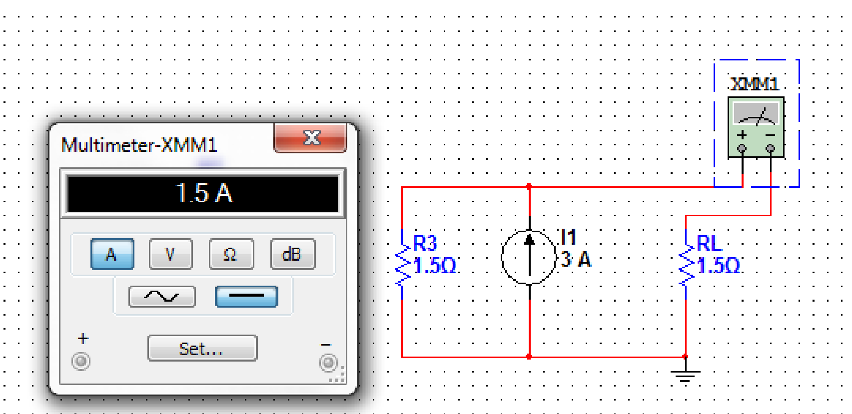

Figure 8

As you can see from Figure 8, our simulation matches the results that we obtained by using superposition. We obtain a current of 1.5A when we analyze the current source's contribution to the power dissipated in R_L. Since the resistors are connected in parallel, there is current division and due to the fact that both of the resistors have the same value, the current will split evenly between the two resistors making the current across R_L $\frac{3}{2}$A.

2(b). Remove the current source by treating it as an open circuit. Find the value of RL that maximizes the power dissipated in RL. Explain why this value is the same, or different, than the answer to (a). Justify your conclusion whether it should be the same or different.

EXPLANATION

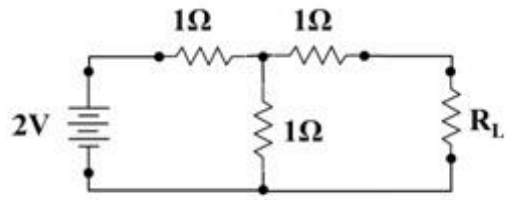

Figure 9

First we must redraw the circuit as you see that we did in Figure 9. This way it is easier to visualize the circuit at hand. We will follow all of the procedures detailed above in order to find R_{TH}, which will again be $\frac{3}{2}\Omega$.

After doing this, we have the exact circuit from Figure 4. We solve for I again in order to use this value in the formula for power $P=I_2R_L$. We use KVL to solve for I, which we find to be $I=\frac{2}{1.5+1.5}=\frac{2}{3}$. Now we use substitution to find the power dissipated in R_L. We use the following equation to find power: $P=I^2R_L=(\frac{4}{9})(\frac{3}{2})=\frac{2}{3}W$

We know that this value should be consistent with the value obtained by analyzing the power contribution by the voltage source. We say this due to the fact that what was asked of us in part b is the part of the theory of superposition which was performed in part a to obtain the power contribution due to the voltage source. The theory of superposition states that to find the contribution of a voltage source, we must find the Thevenin resistance, short any other voltage sources and open current sources, then analyze the contribution of that particular source. This is exactly what we did in part b and also in part a, thus the results should be consistent.

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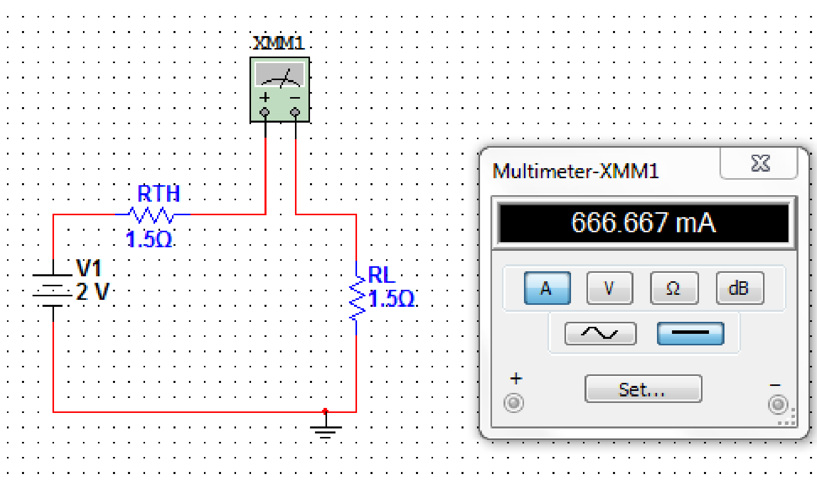
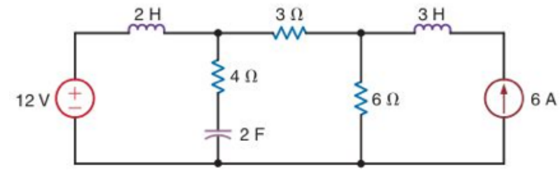


Figure 10

As you can see from Figure 10, our simulation matches the results that we obtained by using superposition. We obtain a current of $\frac{2}{3}$A when we analyze the voltage source's contribution to the power dissipated in R_L. Since the resistors are connected in series, current throughout the circuit is the same I = $\frac{2}{3}$A.

3. Find the energy stored in the 2F capacitor and 3H inductor. (Hint: find all voltages and currents first.)



EXPLANATION

First we state that in order to analyze this circuit, we will assume steady state conditions when t goes to infinity, the inductors are short circuited and the capacitor is open.

Next, we will redraw the circuit so that we can more easily understand the circuit that we will be dealing with. With the inductors now shorted and the capacitor open the circuit looks as shown in Figure 11.

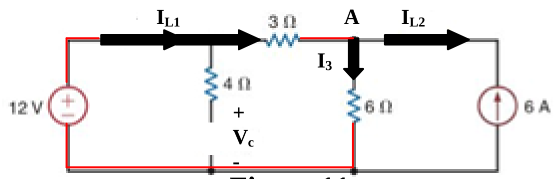


Figure 11

At $t=\infty$ the capacitor becomes an open circuit and the voltage across it is equivalent to the voltage source. We obtain the following KCL equations at node A:

KCL @ A:

$$-I_{l1}+I_{l2}+I_3=0$$
$$I_3=I_{l1}-I_{l2}$$

We can use these equations in conjunction with other equations that we can derive from the circuit.

Now we will analyze the loop highlighted in red to find more useful information.

KVL @ red loop:

$$-12V+I_{l1}(3)+I_3(6)=0$$
$$-12V+I_{l1}(3)+(I_{l1}-I_{l2})(6)=0$$
$$3I_{l1}+(I_{l1}+6)(6)=12V$$
$$9I_{l1}+36=12$$
$$I_{l1}=\frac{-24}{9}$$

From Figure 11, we notice that I_{L2} has the same magnitude as the current source but opposite direction. Due to the fact that we know I_{L1} and I_{L2}, we can find I_3.

$$I_3=I_{l1}-I_{l2}$$
$$I_3=\frac{-24}{9}+\frac{54}{9}$$
$$I_3=\frac{10}{3}$$

Now that we know the currents of the circuit, we can use the equations for energy stored formulas. The energy stored in capacitors follows the equation below:

$$W_{c1} = \frac{1}{2}C(V)^2$$

The energy stored in inductors follows the equation below:

$$W_{l2} = \frac{1}{2}L_2(I_{l2})^2$$

We use these equations and plug in the values according to the circuit in order to find the energy stored in the 3H inductor and the capacitor.

We get the following:

$$W_{c1} = \frac{1}{2}(2F)(12)^2$$

$$W_{c1} = 144J$$

$$W_{l2} = \frac{1}{2}(3H)(-6)^2$$

$$W_{l2} = 54J$$

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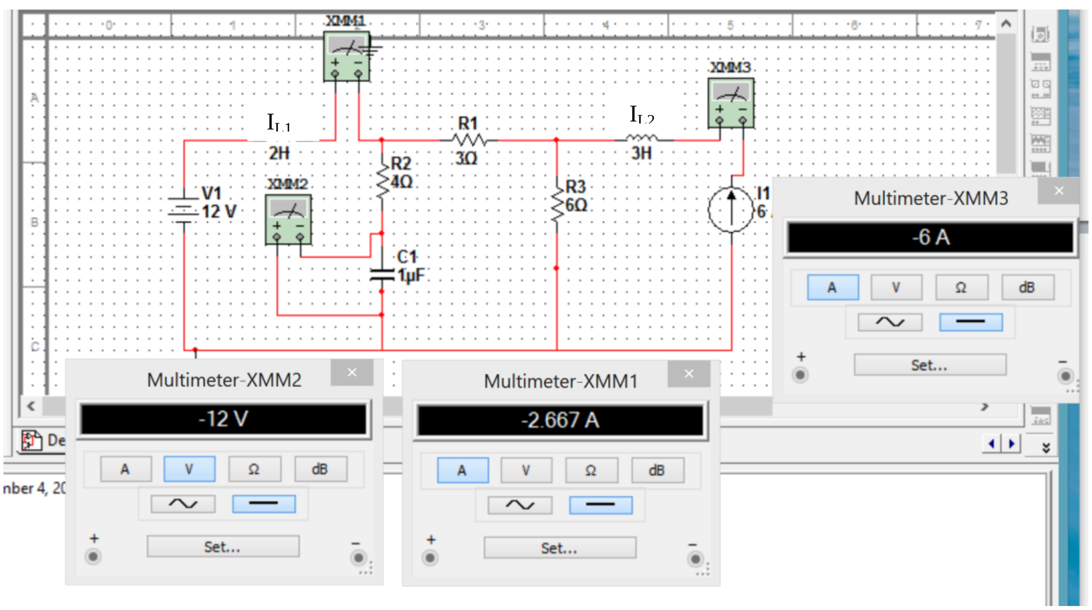

Figure 12

As you can see from Figure 12, our simulation matches the result. Figure 12 shows that the current through the L1 is $\frac{-8}{3}$ as we expected. Also, we find that I_{L2} is -6A and V_c is 12V as we expected them to be.