Name: Mohammad Hakim

Problem	Points	Score
1	30	
2(a)	30	
2(b)	10	
3	30	
Total	100	

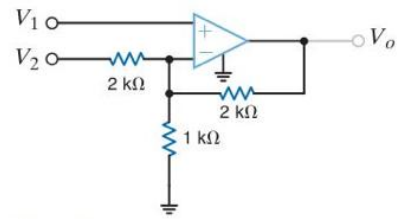
Notes:

(1) The exam is closed books and notes except for one double-sided sheet of notes.

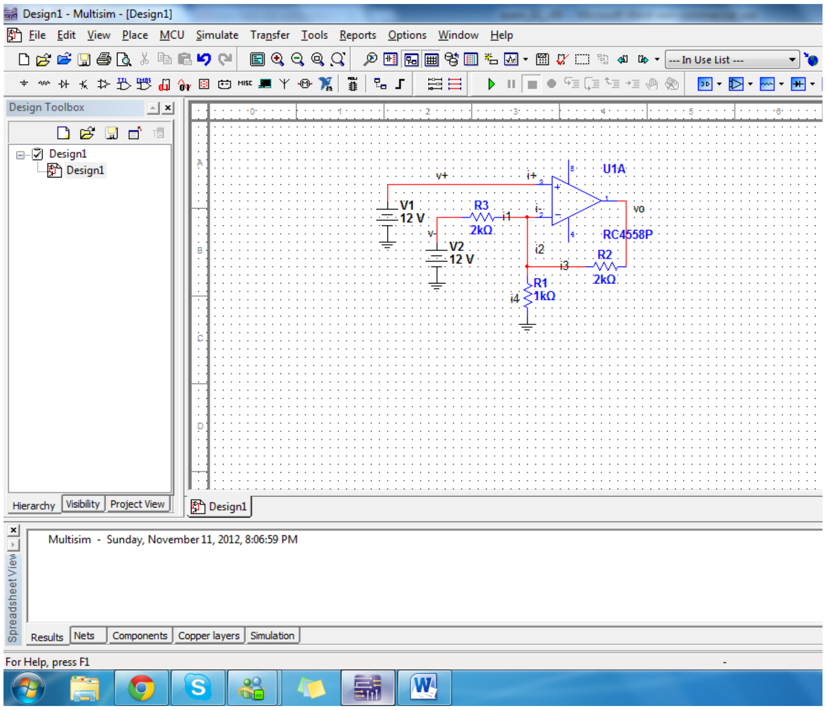
(2) Please indicate clearly your answer to the problem. Circle your answers.

(3) The details of your solutions are more important than the answers. Please explain your solutions clearly and include as many details as possible.

1. Derive an expression for V_o as a function of V_1 and V_2.



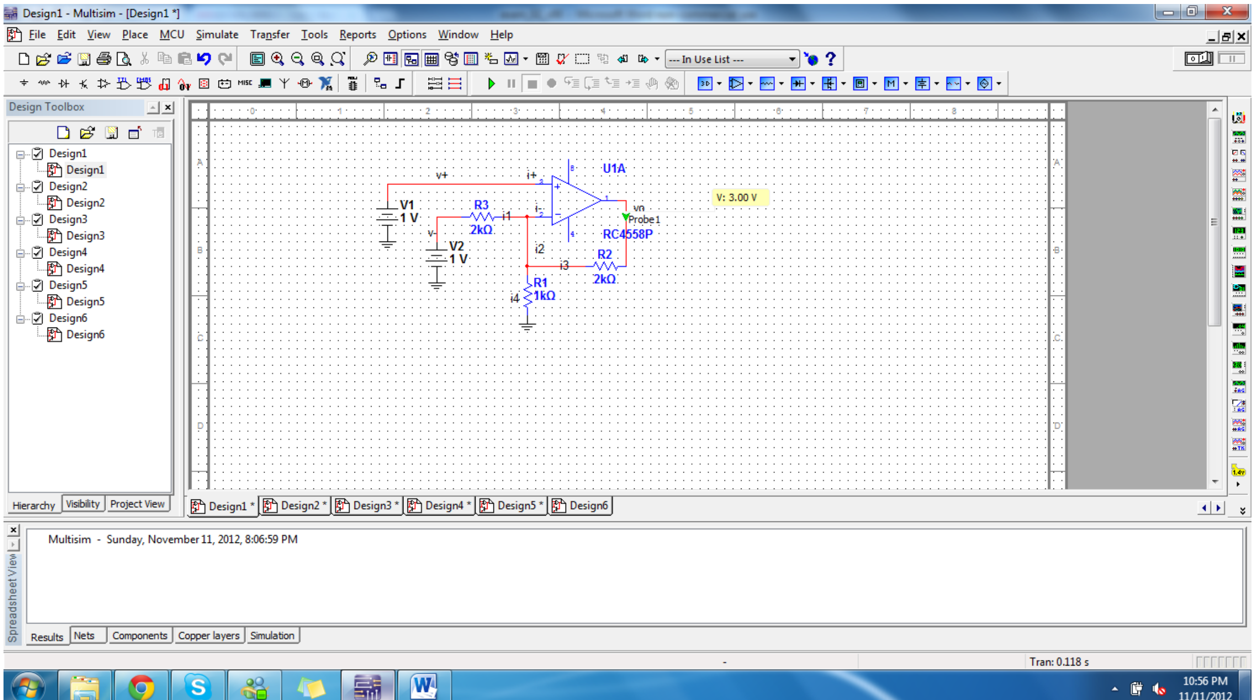
Redraw circuit and label it:

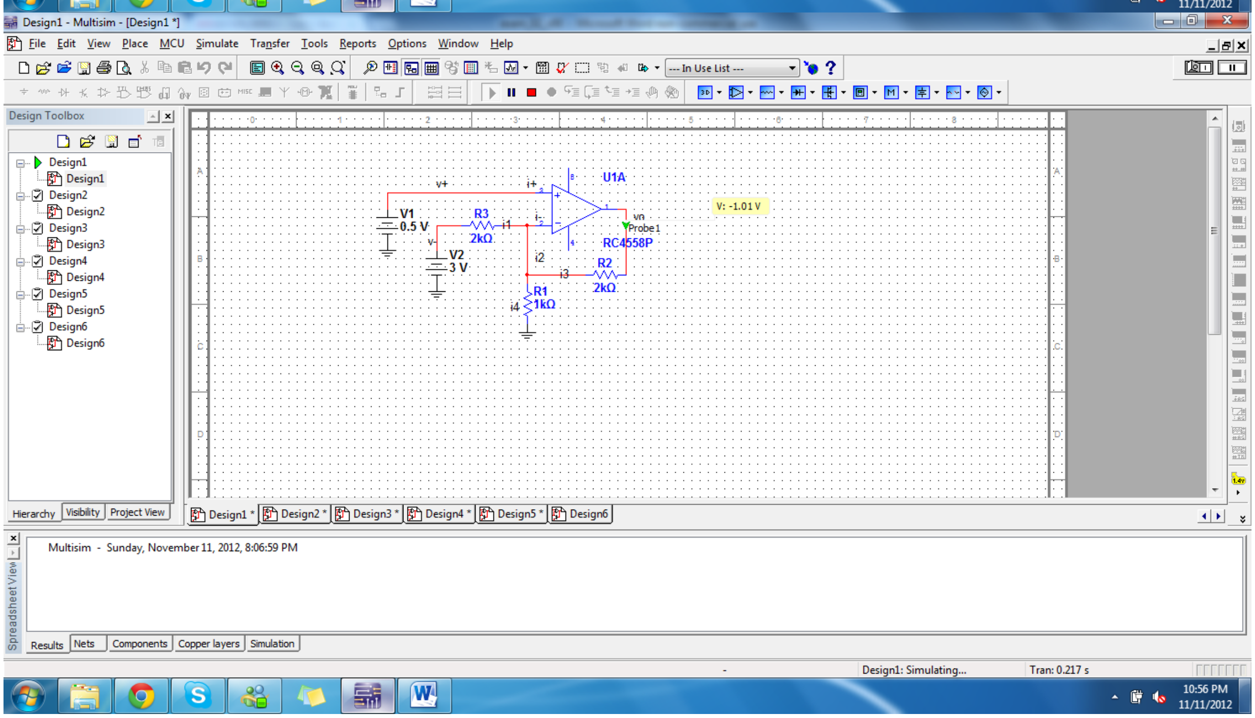


We first write the conditions then apply a KCL and substitute.

$$v+=v-$$
$$i+=i-$$
$$i_1=i_3$$
$$thus\ i_2+i_3=i_4$$
$$KCL:$$
$$\frac{v2-v-}{2k}+\frac{vo-v-}{2k}=\frac{v-}{1k}$$
$$\frac{v2}{2k}-\frac{v-}{2k}+\frac{vo}{2k}-\frac{v-}{2k}=\frac{v-}{1k}$$
$$\frac{v2}{2k}+\frac{vo}{2k}=2\frac{v-}{1k}\ where\ v-=v1$$
$$\frac{vo}{2k}=\frac{2v-}{1k}-\frac{v2}{2k}$$
$$vo=2k\left(\frac{2v-}{1k}-\frac{v2}{2k}\right)$$
$$vo=4v--v2$$
$$\mathbf{vo=4v1-v2}$$

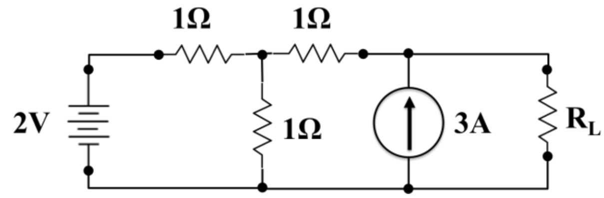
Check in multisim:



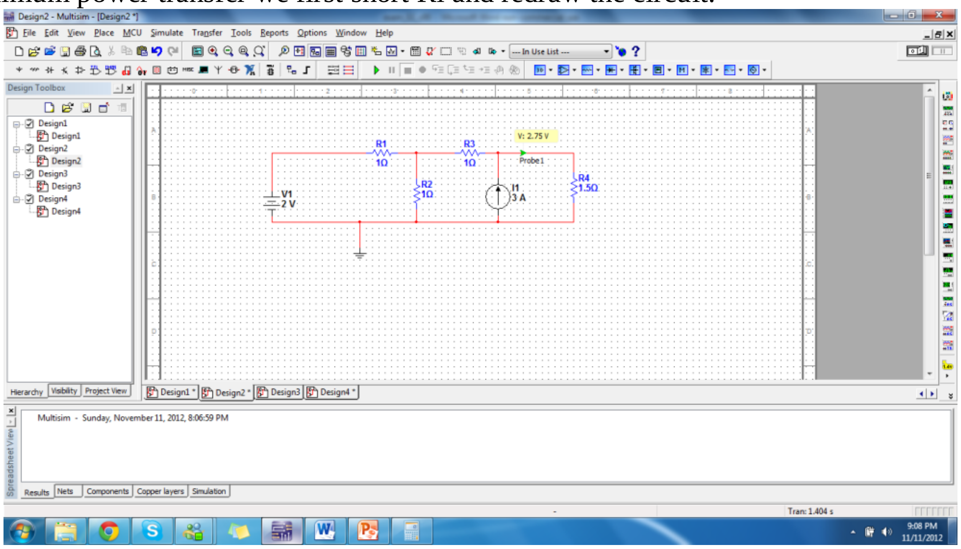


SO LU TION APPLIES

2. (a) Using superposition, **f**ind the value of R_L that maximizes the power dissipated in R_L. Compute the portion of this power due to the voltage source only, and the portion due to the current source.

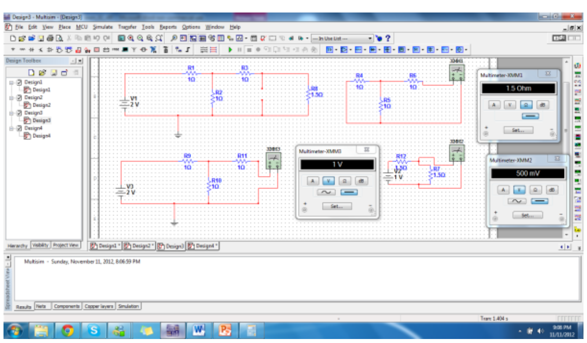


To find maximum power transfer we first short Rl and redraw the circuit.



We can now apply super position:

We take out the current source and try to solve for the voltage across the resistor



To find the voltage across the load we apply thevenin equivalent. By shorting the voltage source we solve $R_{eq} = \frac{1\times1}{1+1} + 1 = 1.5\ \Omega$(also R load for maximum power transfer theory)

Using a voltage divider equation we can find the voltage for the thevenin equivalent:

$$V_{eq} = \frac{1}{1+1} \times 2v = 1v$$

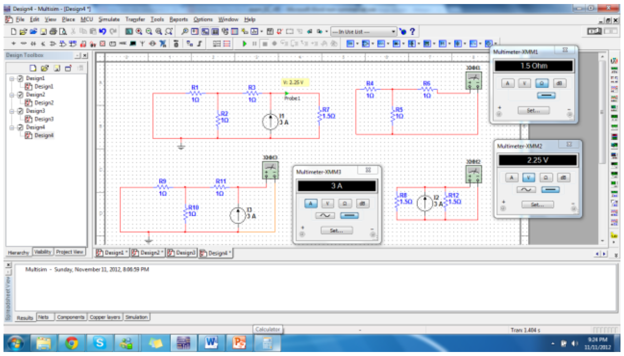
Now we can find V'_{output} across R load by using voltage divider again

$$V'_{output} = \frac{1.5}{1.5+1.5} \times 1v = 500mv$$

Now we can calculate power:

$$P_{voltage} = \frac{V^2}{R} = \frac{500v^2}{1.5\Omega} = 166.667mW$$

We take out the voltage source and try to solve for the voltage across the resistor



To find the voltage across the load we apply norton equivalent. By opening the current source we solve $R_{eq} = \frac{1\times1}{1+1} + 1 = 1.5\ \Omega$(also R load for maximum power transfer theory)
Using a current divider equation we can find the current for the norton equivalent:

$$I_{eq} = \frac{1.5}{1||1+1} \times 3A = 3A$$

Now we can find V''_{output} across R load by using current divider again

$$I_{eq} = \frac{1.5}{1.5+1.5} \times 3A = 1.5A$$
$$V''_{output} = 1.5 \times 1.5 = 2.25v$$

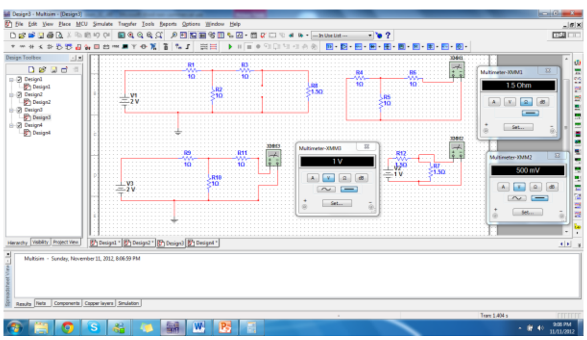
Now we can calculate power:

$$P_{current} = I^2R = 1.5^2 \times 1.5 = 3.375W$$

Now summing up:

$V_{output} = V'_{output} + V''_{output} = .5 + 2.25 = 2.75v$ as shown in the first figure above.

2(b). Remove the current source by treating it as an open circuit. Find the value of RL that maximizes the power dissipated in RL. Explain why this value is the same, or different, than the answer to (a). Justify your conclusion whether it should be the same or different.



If we remove the current source we are left with the part of the question that deals with the voltage source. Shown below:

We take out the current source and try to solve for the voltage across the resistor

To find the voltage across the load we apply thevenin equivalent. By shorting the voltage source we solve $R_{eq} = \frac{1\times 1}{1+1} + 1 = 1.5\ \Omega$(also R load for maximum power transfer theory)

Using a voltage divider equation we can find the voltage for the thevenin equivalent:

$V_{eq} = \frac{1}{1+1} \times 2v = 1v$

Now we can find V'_{output} across R load by using voltage divider again

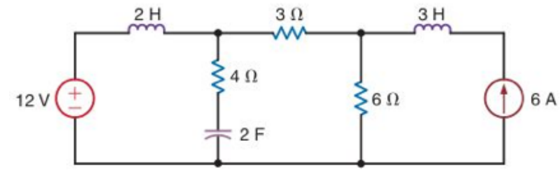
$$V'_{output} = \frac{1.5}{1.5 + 1.5} \times 1v = 500mv$$

Now we can calculate power:

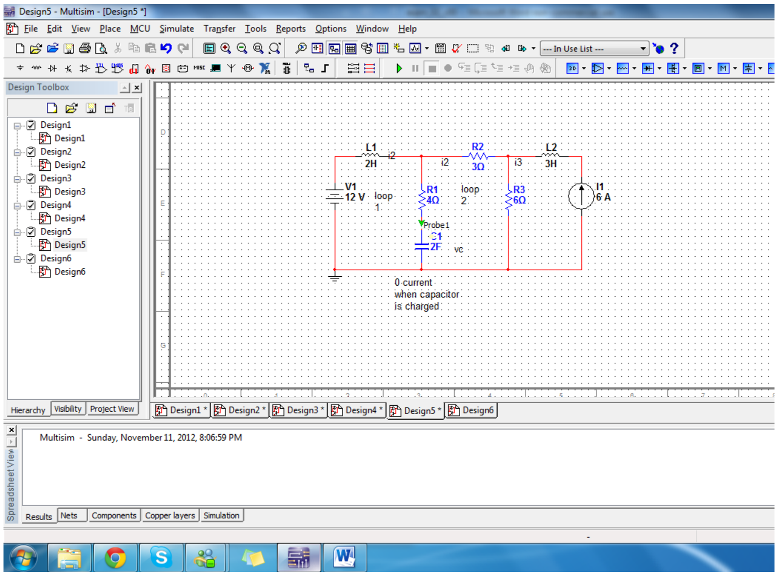
$$P_{voltage} = \frac{V^2}{R} = \frac{500v^2}{1.5\Omega} = 166.667mW$$

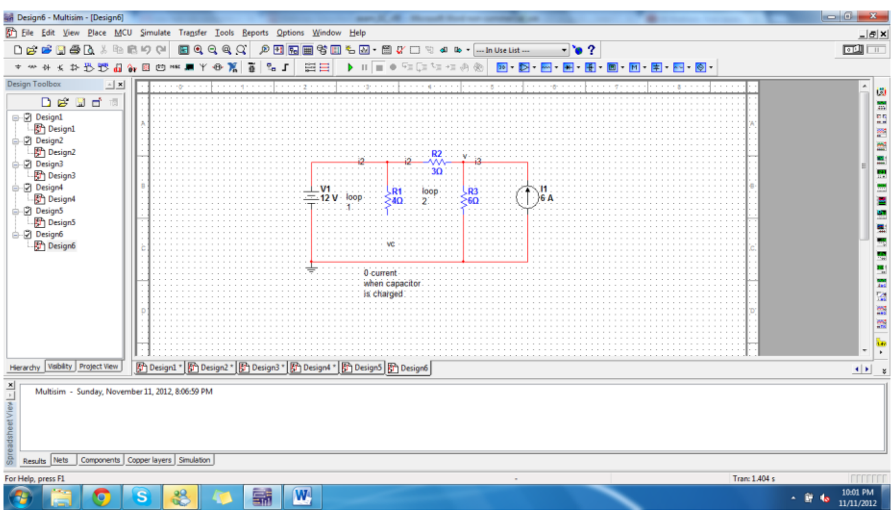
Notice that Req is still the same, that is why the value of R load is also going to be the same.
The reason for that is that when we remove The current source we only are changing the orientation of a source that will be removed anyways when we attempt to calculate Req for the circuit, thus no change is observed.

3. Find the energy stored in the 2F capacitor and 3H inductor. (Hint: find all voltages and currents first.)



I redraw the circuit and label it:





Short the inductors and open the capacitors to begin steady state analysis and redraw the circuit.

Apply KCl at node v:

$$\frac{12-v}{3} = \frac{v}{6} - 6$$

$$4 - \frac{v}{3} = \frac{v}{6} - 6$$

$$v = 20$$

$$I_2 = \frac{12v - 20v}{3\Omega} = \frac{-8}{3}A$$

$$I\ mesh1 = I_1$$

$$I_1 = I_2$$

Type equation here.

Apply KVL at mesh 1:

$$-12v + 4(I_1 - I_2) + V_c = 0$$

$$\text{-12v+4}(\tfrac{-8}{3} + \tfrac{8}{3}) + V_c = 0$$

$$V_c = 12v$$

$$W_{c=2F} = \frac{1}{2}(2F)(12v)^2 = 144w$$

The current through L=2H is $I_2 = \frac{-8}{3}A$ so

$$W_{L=2H} = \frac{1}{2}(2H)\left(\frac{-8}{3}A\right)^2 = 7\frac{1}{9}w$$

The current through L=3H is $I_3 = -6A$

$$W_{L=3H} = \frac{1}{2}(3H)(-6A)^2 = 54w$$

We can verify that our solution for currents and voltages is right by using Dc operating point analysis