

question_3

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(3) Consider two RVs that are uncorrelated:

$$H(X, Y) = H(X) + H(Y) = H_i$$

Does the $H(X, Y)$ increase or decrease if the variables become more correlated?

Answer:

$H(X, Y)$ decreases with increase in correlation.

Consider the following definition of mutual information $I(X; Y)$, where $H(X)$ and $H(Y)$ are the marginal entropies, $H(X|Y)$ and $H(Y|X)$ are the conditional entropies, and $H(X, Y)$ is the joint entropy of X and Y :

$$H(X, Y) = I(X; Y) + H(X|Y) + H(Y|X)$$

If we consider the mutual information $I(X; Y)$ to be a measure of correlation, then as this quantity increases it would appear that the mutual entropy should increase.

However, we must remember that any increase in the mutual information $I(X; Y)$ yields a corresponding decrease in both conditional entropies $H(X|Y)$ and $H(Y|X)$ - so any increase ϵ in mutual information $I(X; Y)$ yields a corresponding decrease ϵ in both $H(X|Y)$ and $H(Y|X)$.

Thus, for any increase ϵ in mutual information $I(X; Y)$ yields a corresponding decrease in $H(X, Y)$ of $2\epsilon - \epsilon = \epsilon$