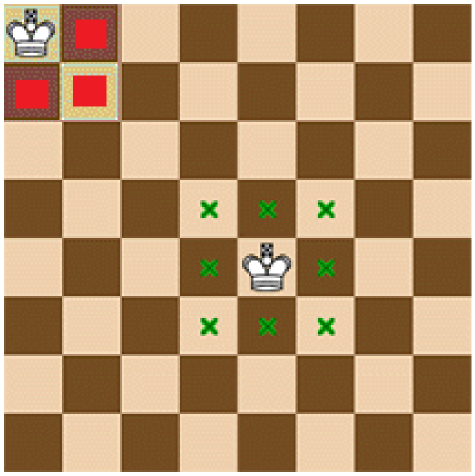
Example 4.3.1

The King can move one step in all directions.

At corners

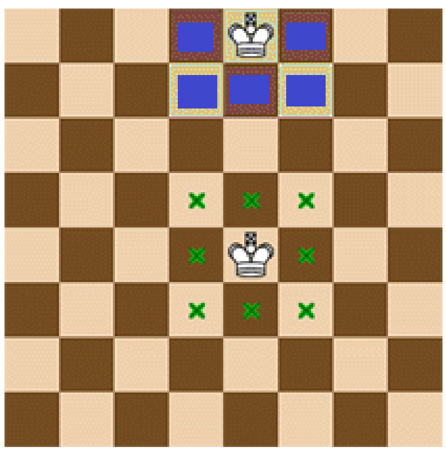


No. of possible moves at **each** corner = 3 (**number of edges emanating from each node = E_1**)

Total number of corners = 4

Total number of possible moves at **all corners** = 4×3 = 12

At edges (excluding corners):



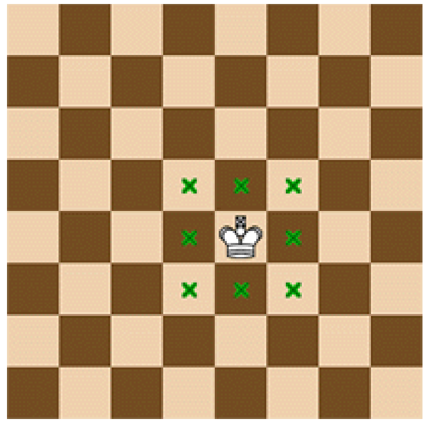
Total possible moves at each edge = 5 (**number of edges emanating from each node = E_2**)

Total edges on one side = 6

Total edges = 6×4 = 24 (because we have 4 total sides)

Total possible moves for **all edges** = 5×24 = 120

Elsewhere



Possible moves at each block (except corner and edge) = 8 (**number of edges emanating from each node = E_3**)

Total blocks on chess board = 8×8 = 64

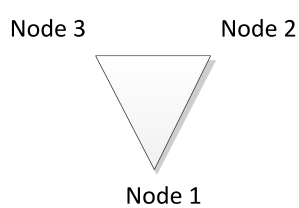
Total blocks (except corners and edges) = 36

Total possible moves at chess board except corners and edges = 36 × 8 = 288

Total possible moves on board = total moves at corners + total moves at edges + total moves elsewhere

Total possible moves on board = 12 + 120 + 288 = 420

E is the total number of possible edges in the graph and is different from total number of possible moves on chess board. Now, for the case of simplicity, consider a random walk on the following triangle



The total number of edges in the above graph is 3 whereas total number of possible moves is 6.

On chess board, total number of possible moves includes the moves towards the forward direction and the moves towards backward direction (similarly, it includes possible move to the right nodes as well as the left nodes). Therefore, total number of possible edges is half the number of possible moves.

Therefore, 2**E** =420

Using equation 4.41, we have:

$$H(\mathcal{X}) = \log_2 2E - H\left(\frac{E_1}{2E}, \frac{E_2}{2E}, \frac{E_3}{2E}\right)$$

$$H(\mathcal{X}) = \log_2 420 - H\left(\frac{3}{420}, \frac{5}{420}, \frac{8}{420}\right)$$

$$H(\mathcal{X}) = \log_2 420 - \left(-4 \times \frac{3}{420} \times \log_2 \frac{3}{420} - 24 \times \frac{5}{420} \times \log_2 \frac{5}{420} - 36 \times \frac{8}{420} \times \log_2 \frac{8}{420}\right)$$

where 4, 24 and 36 were the total number of boxes having 3, 5, and 8 possible moves respectively.

Thus,

$$H(\mathcal{X}) = 2.7658$$

Now, consider an infinite chess board. Each node will have 8 connecting nodes. Let's calculate the entropy rate (for the random walk of a King) on a block of 8×8 nodes:

Total nodes = 8×8 = 64

Possible moves at each node = 8

Total possible moves = 64×8 = 512

Calculating entropy rate:

$$H(\mathcal{X}) = \log_2 512 - \left(-64 \times \frac{8}{512} \times \log_2 \frac{8}{512}\right) = 3 = \log_2 8$$

which is same as the entropy rate for 16×16 block on an infinite chess board calculated as:

Total nodes = 16×16 = 256

Possible moves at each node = 8

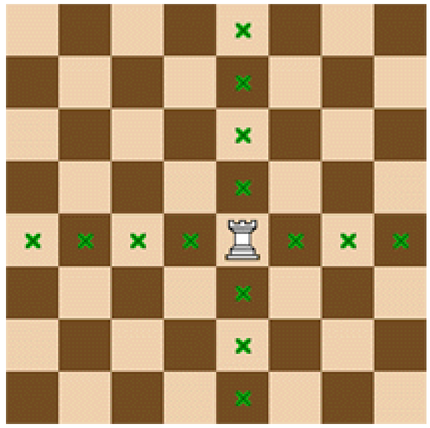
Total possible moves = 256×8 = 2048

$$H(\mathcal{X}) = \log_2 2048 - \left(-256 \times \frac{8}{2048} \times \log_2 \frac{8}{2048}\right) = 3 = \log_2 8$$

Thus, entropy rate of an infinite chess board is $\log_2 8$.

For King, the entropy rate on our real chess board becomes $0.922 \times \log_2 8$ approximately. The factor 0.922 is due to edge effects.

For rook



There are 14 possible movements of rook at each node on the chess board.

Total possible movements = 64×14 = 896

Total blocks on chessboard = 64

$$H(\mathcal{X}) = \log_2 896 - \left(-64 \times \frac{14}{896} \times \log_2 \frac{14}{896}\right) = 3.8074 = \log_2 14$$

It is interesting that a rook has 14 possible moves at each block and its entropy rate is $\log_2 14$.