Problem: for a Gaussian distribution with a given variance, v1, what is a uniform distribution with the same entropy?

Solution: For a Uniform Distribution from -a to a, its differential entropy

$$h(X) = -\int_{-a}^{a} \frac{1}{2a} \log \frac{1}{2a} dx = \log 2a$$

For a Normal distribution, let $X \sim \phi(x) = \left(1/\sqrt{2\pi\sigma^2}\right) \times e^{-x^2/2\sigma^2}$, its differential entropy is

$$h(\phi) = \frac{1}{2} \log 2\pi e\sigma^2$$

If the two distribution has the same differential entropy is

$$\frac{1}{2} \log 2\pi e\sigma^2 = \log 2a$$

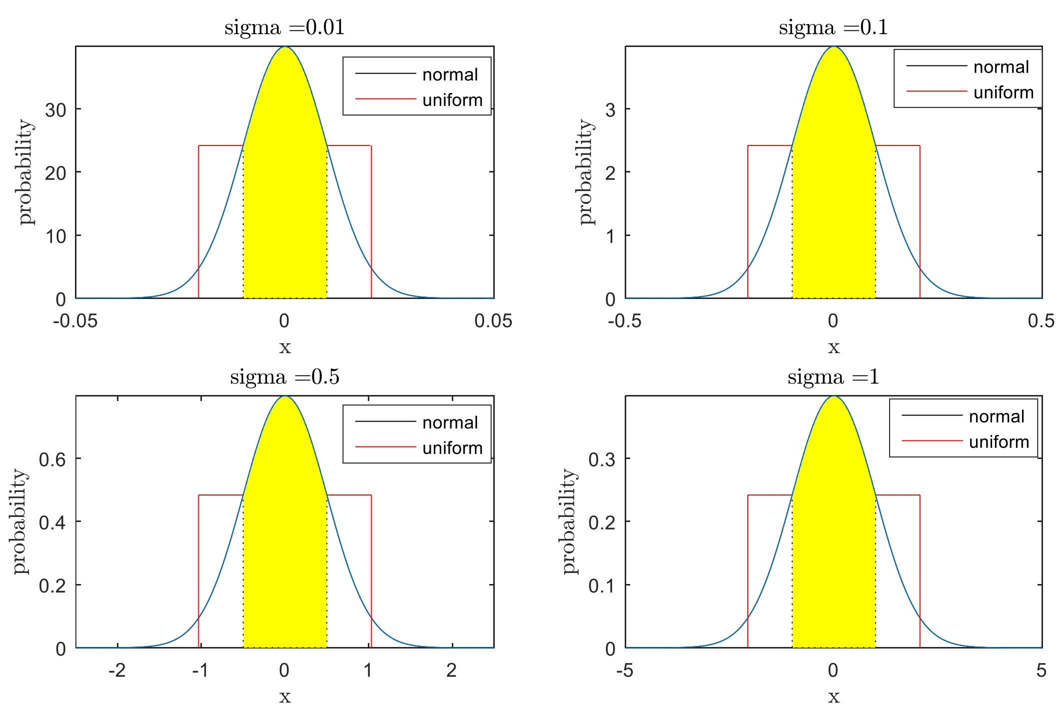
$$\Downarrow$$

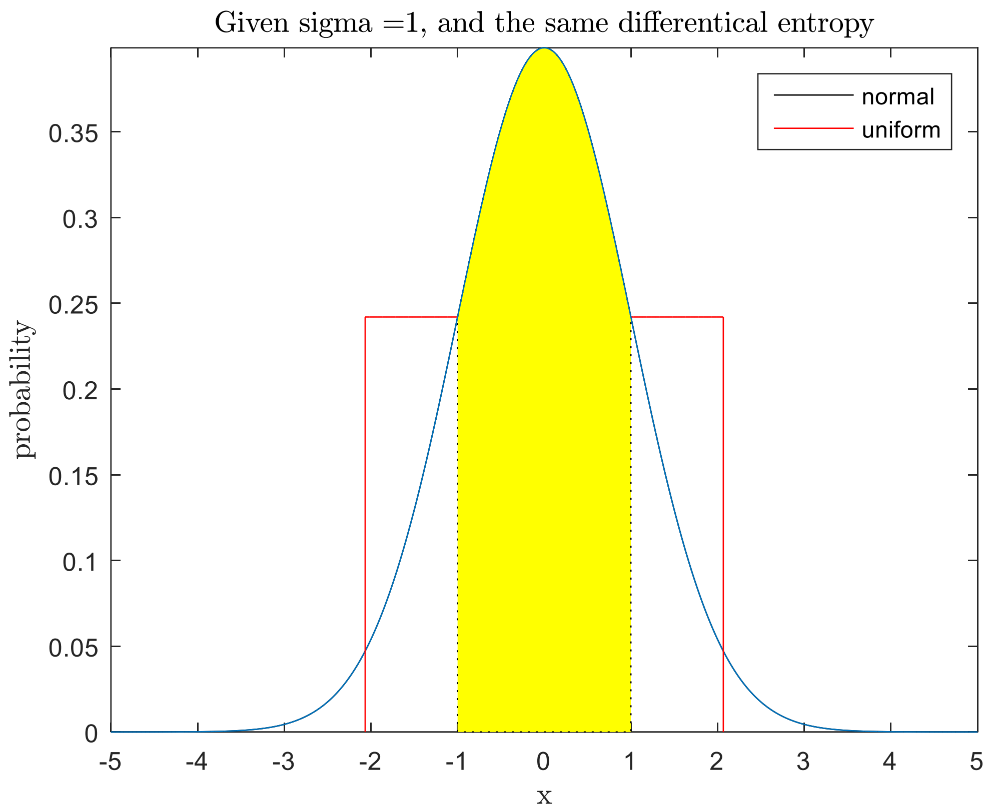
$$2a = 2^{\frac{1}{2}\log 2\pi e\sigma^2}$$

$$2a = \sigma\sqrt{2\pi e}$$

$$a = \frac{\sigma\sqrt{2\pi e}}{2}$$

If the two distribution have the same entropy, the pdf of two distributions are depicted as follows. The yellow part is the range of [-sigma, sigma]. We could see that the relationship between the pdf of Gaussian and Uniform look similar when sigma changes, given the same differential entropy.





The above figure shows that given the same differential entropy, uniform distribution is outside the range of [-sigma, sigma]. Almost has the same range of [-2*sigma, 2*sigma], and it is inside of [-3*sigma, 3*sigma].

$$\Pr(\mu - \sigma \leq X \leq \mu + \sigma) \approx 0.6827$$
$$\Pr(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx 0.9545$$
$$\Pr(\mu - 3\sigma \leq X \leq \mu + 3\sigma) \approx 0.9973$$