Fast Gaussian Evaluations in Large Vocabulary Continuous Speech Recognition

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Organization of Presentation

* Motivation for fast Gaussian computation (FGC)
* Bucket Box Intersection (BBI) algorithm and issues related to the BBI algorithm
* Extended BBI (EBBI) algorithm
* Comparison between the EBBI and BBI algorithms
* Comparison between the EBBI algorithm and Gaussian Clipping (GC) technique
* Experimental results and analysis
* Conclusions and future work

Motivation

* State likelihood computations in the statistical modeling component of the speech recognition systems contribute 50%-80% to the computational load

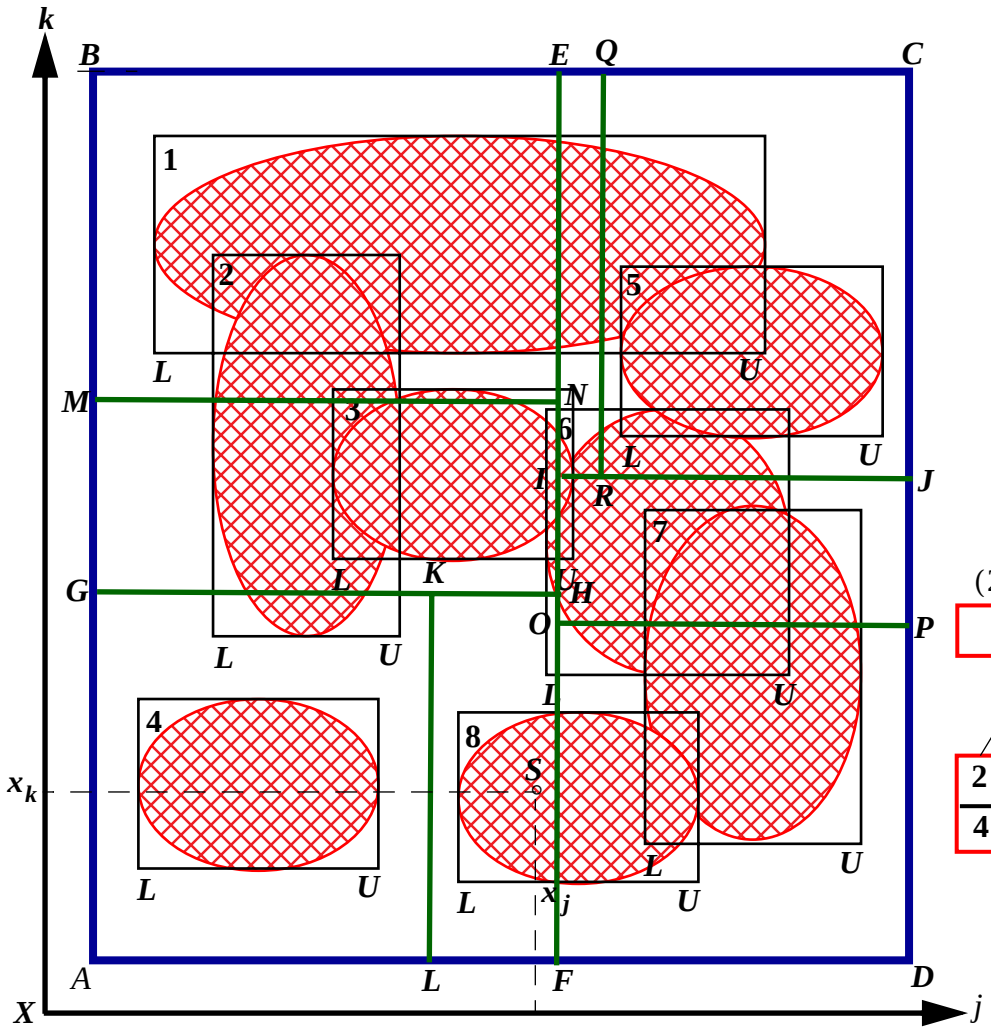
* The likelihood computation is dominated by a few (significant) Gaussian components and can be computed solely from these Gaussian components without a significant increase in the system WER

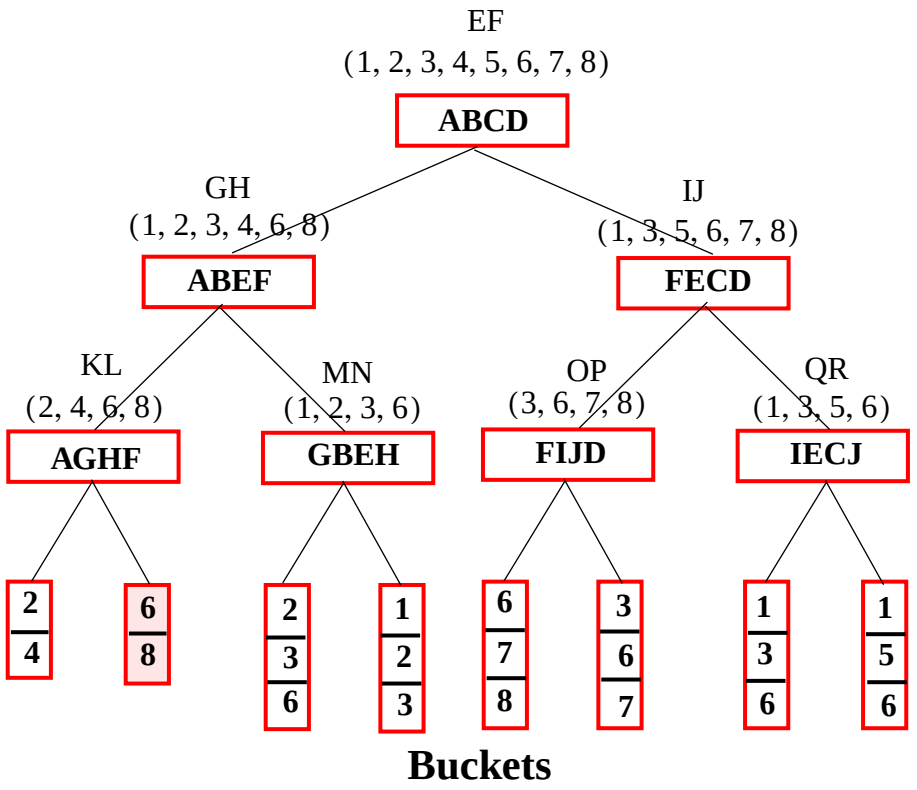
* Need an efficient way to find the most significant Gaussians for the likelihood computation

BBI Algorithm (I)

* Use of a pre-computed multi-dimensional decision tree (k-d tree) to determine the set of the most significant Gaussians
* Represents all the Gaussians in the mixture by using the Gaussian boxes and uses the Gaussian boxes to build the k-d trees
* Restrict the likelihood computation to the Gaussians with boxes that contain the vector
* Algorithm parameters — tree depth and relative threshold

BBI Algorithm (II)





Issues Related to the BBI Algorithm

Optimization criterion:

- Optimization criterion used by the BBI algorithm requires the selection of a balanced hyperplane that produces a minimum number of Gaussian splits

- In practical applications there may be more than one collection of hyperplanes that produce the same (minimum) number of Gaussian splits

Mixture weight re-normalization:

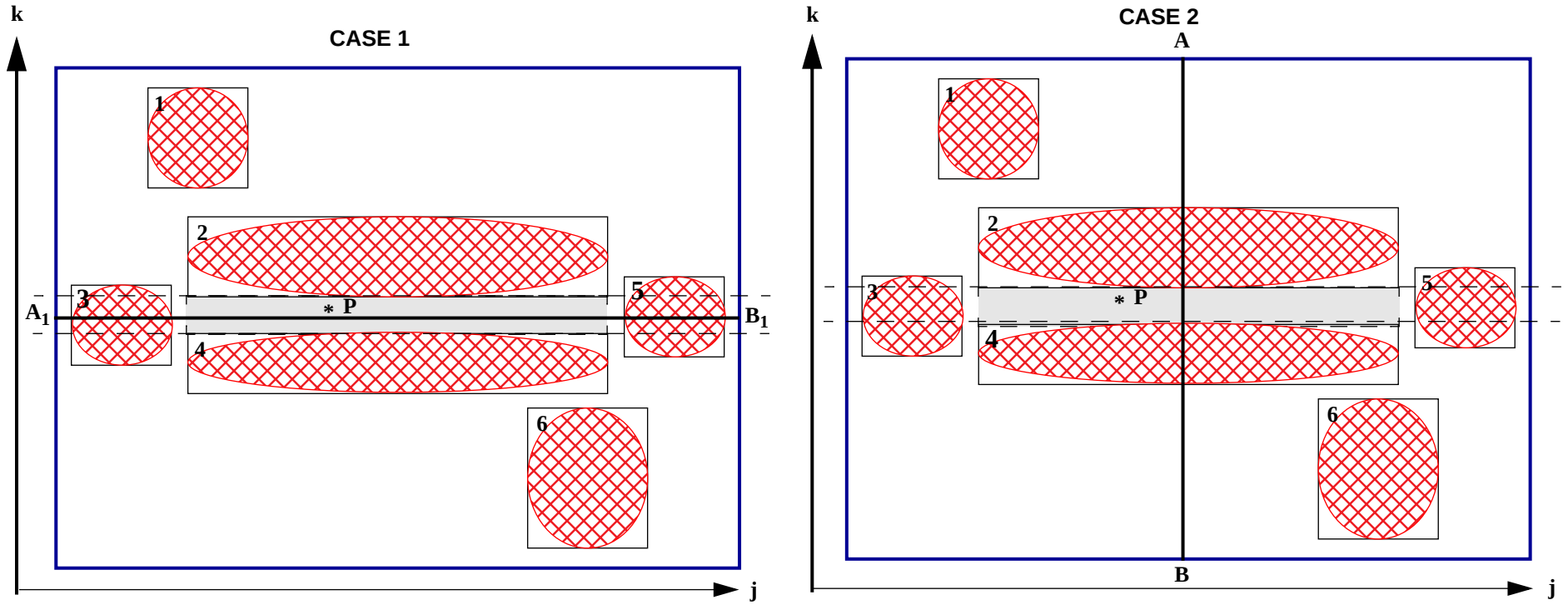
- BBI algorithm restricts the likelihood computation to the Gaussians that contain the feature vector but it doesn't re-normalize the mixture weights

EBBI Algorithm

A Modified Optimization Criterion (I)

* The modified optimization criterion for the multiple minima in Gaussian splits includes following steps:
  * Finding the variance of those coordinate axes whose hyperplanes produce a minimum number of Gaussian splits
  * Choosing the hyperplane corresponding to the coordinate axis with the highest variance

A Modified Optimization Criterion (II)



* Given the positions and forms of the Gaussians in the region, Gaussians 2 and 4 can contain the vector *P* (shown in the figure) with an equal probability

A Modified Optimization Criterion (III)

* Hyperplanes AB and A_1B_1 give a balanced division and produce the same (= 2) number of Gaussian splits
* A k-d tree obtained by using the hyperplane A_1B_1 produces a higher approximation error
* Under the framework of the division hyperplane AB, Gaussians 2 and 4 both contain the vector P
* A k-d tree obtained by using the hyperplane AB produces a lower approximation error
* In general, the hyperplane AB produces a lower approximation error for all the vectors in the shaded region

Mixture Weight Re-normalization (I)

* The log-likelihood of a feature vector in a k-dimensional space is $L = \sum_{m=1}^{M} \log(w_m \cdot P_m)$, where, w_m is the weight of the m^{th} Gaussian and P_m is the pdf of the m^{th} Gaussian in a mixture of M Gaussians

* If a feature vector lies in a bucket containing the Gaussians l and $l + p$, the log-likelihood of the vector is: $L = \log(w_l \cdot P_l) + \log(w_{l+p} \cdot P_{l+p})$

Mixture Weight Re-normalization (II)

* Using normalized weights, the log-likelihood is:

$$L' = \log\left(\frac{w_l}{w_l + w_{l+p}} \cdot P_l\right) + \log\left(\frac{w_{l+p}}{w_l + w_{l+p}} \cdot P_{l+p}\right)$$

* If $w_l + w_{l+p} = C$, $L' = L - 2 \cdot \log C$

* Since $C \leq 1$ or, $\log C \leq 0$, $L' \geq L$

* Use of re-normalized weights results in higher log-likelihoods, which in turn leads to an improved system performance

Selection of a K-d Tree

* There are several options for how we apply a k-d tree to the HMM model (i) one k-d tree per state, (ii)) a single large k-d tree shared by all the HMM states, or (iii)multiple k-d trees with each tree sharing a subset of the states

* Previous implementations of the BBI algorithm successfully employed the latter two options

* The EBBI algorithm uses one k-d tree for each HMM state, because it results in the lowest approximation error and produces the significant speedups as compared to the other two options

EBBI and BBI Algorithms (I)

* Industry-standard OGI-Alphadigits database (telephone database of 6-word strings) used
* 39-dimensional MFCC features used
* 64-mixture cross-word triphone models used
* K-d trees with a tree depth 6 and an error threshold 0.4 used for the two algorithms
* The baseline system produced a 10.1% WER and used 57.9% of the total CPU time for the Gaussian Computations

EBBI and BBI Algorithms (II)

Algorithm	% of Total CPU Time used by Gaussian Evaluations	WER (%)
EBBI	29.1	10.2
BBI	24.7	11.4

* The EBBI algorithm used the modified optimization criterion, mixture weight re-normalization and state level k-d trees
* The EBBI algorithm produced a significant improvement in WER over the BBI algorithm
* The EBBI algorithm produced only a 1.8% lower speedup than the BBI algorithm (not a significant difference)

EBBI Algorithm and GC Technique

* Gaussian clipping technique is used to generate lower-order mixture models from higher-order mixture models by removing the Gaussians with the lower mixture weights

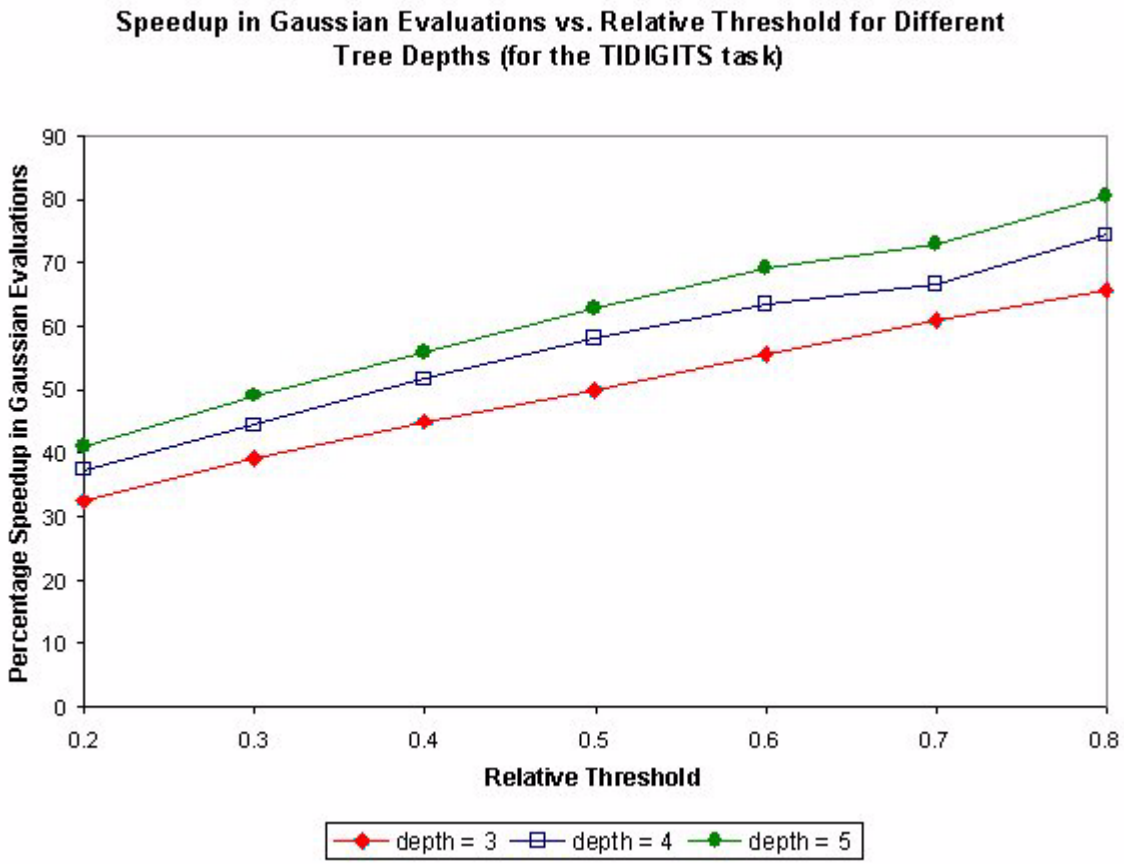
Gaussian speedup (%)	Technique used	WER (%)
25	EBBI	10.3
	GC	11.5
50	EBBI	10.7
	GC	12.2

* For a 50% speedup in the likelihood computation, the EBBI algorithm produced a 12.3% improvement over the GC approach

Performance of the EBBI Algorithm
TIDIGITS Database

* A small database containing continuous digits (vocabulary size of 11)
* The relative threshold used by the algorithm was varied from 0.2 to 0.8 in steps of 0.1
* K-d trees with depth between 3 and 5 were used
* The system which is not using the EBBI algorithm (a No EBBI system) produced a 0.6% WER
* Gaussian evaluations took 24.4% of the total CPU time in this system

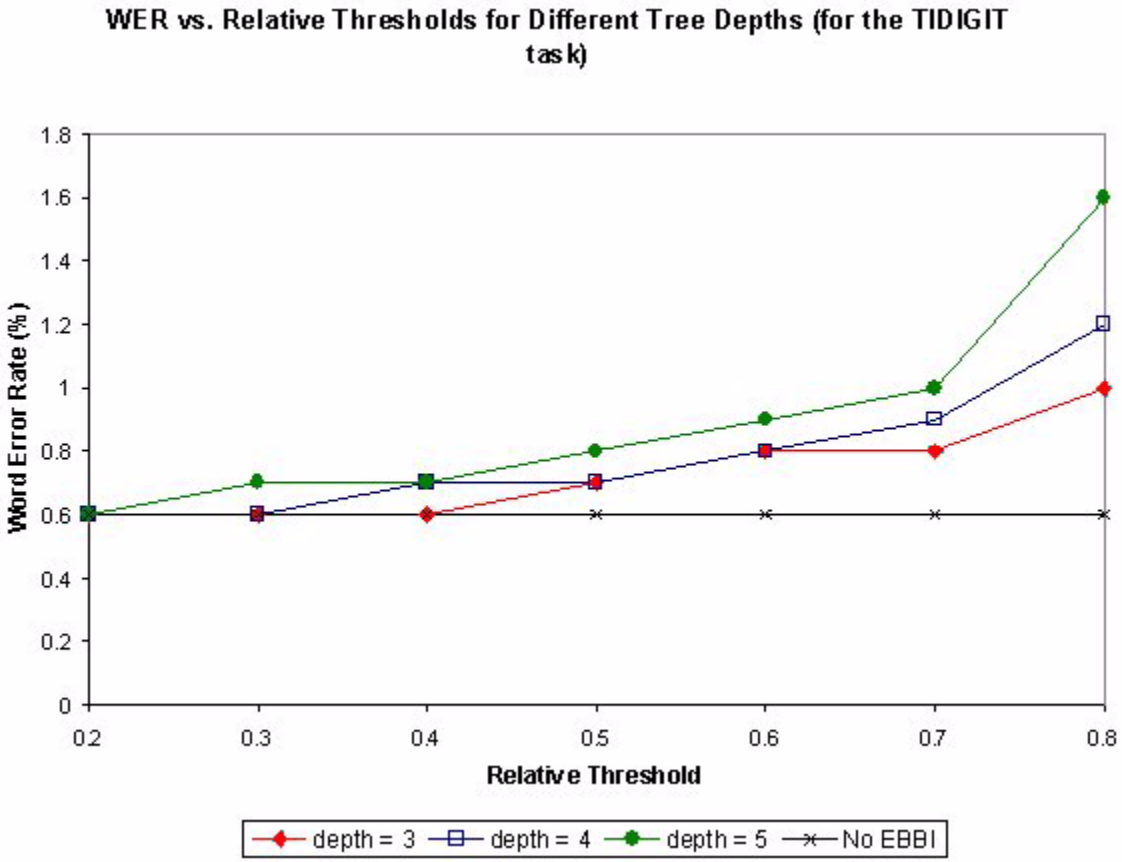
TIDIGITS - Gaussian Speedup



* For a fixed tree depth, the Gaussian speedup increases with an increase in the relative threshold

* For a fixed threshold, the Gaussian speedup increases with an increase in the k-d tree depth

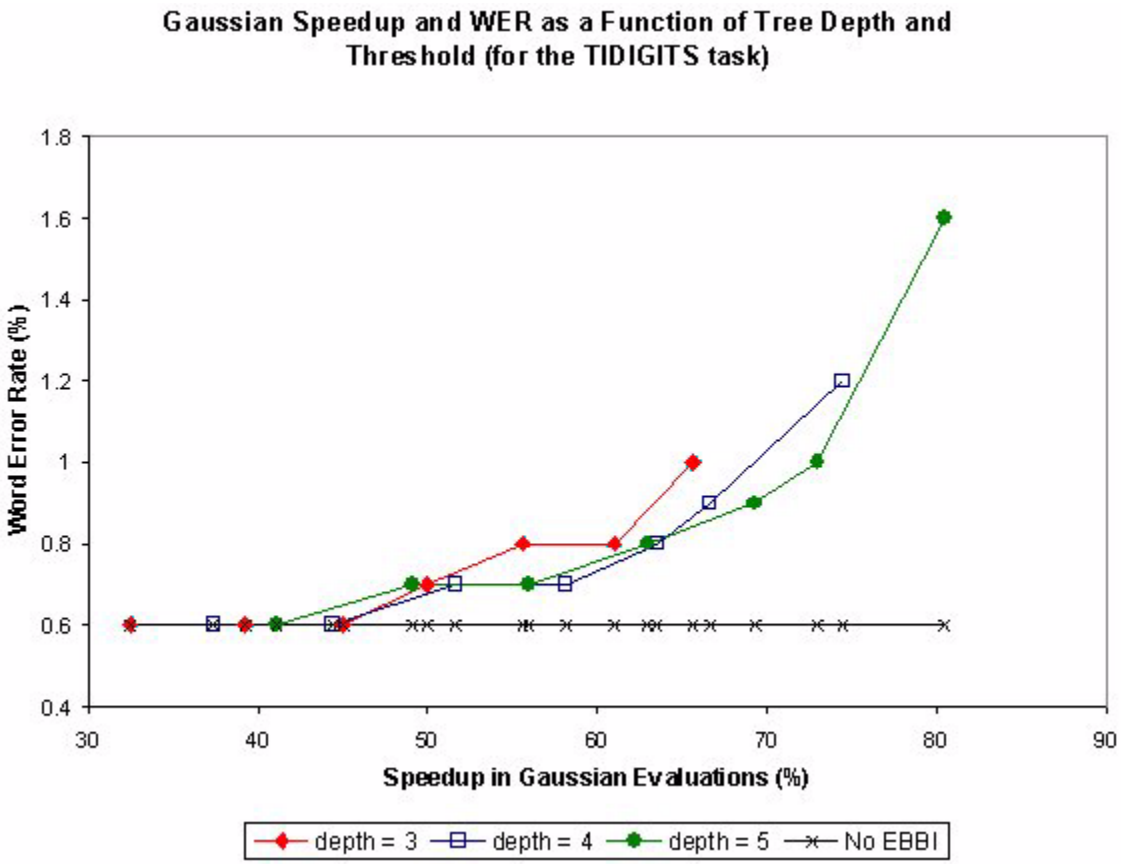
TIDIGITS - WER



* For a fixed tree depth, the WER increases with an increase in the relative threshold

* For a fixed threshold, the WER increases with an increase in the k-d tree depth

TIDIGITS - Speedup and WER



* Best performance — a tree depth of 3 with a relative threshold of 0.4 produced a 45% speedup in score computations without degrading the performance of the system

OGI-AD Database

* Telephone database of 6-word strings
* 39-dimensional MFCC features used
* HMM system used cross-word context-dependent triphone models
* 5-state left-to-right models used
* Used to evaluate the algorithm performance as the function of the number of mixture components — 16, 32 and 64 mixture Gaussians per state used

AD - 16 Mixture Components (I)

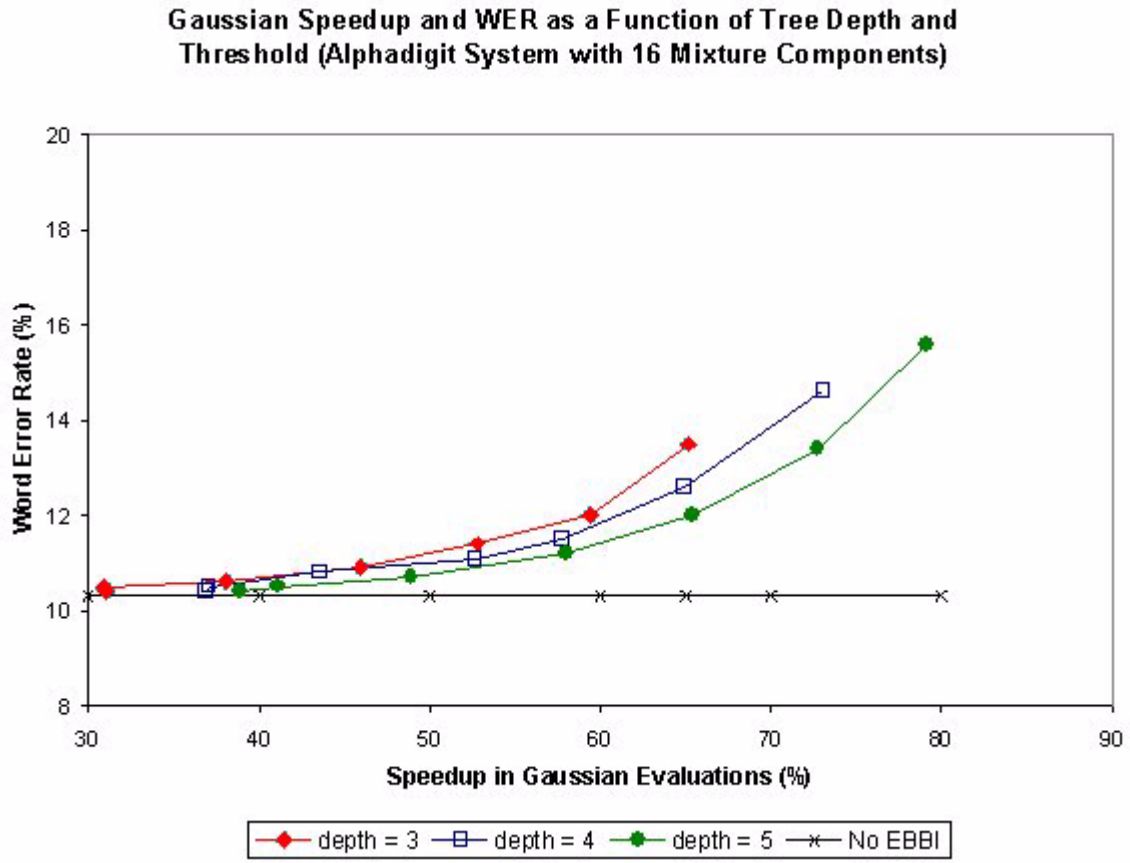
* The relative threshold was varied from 0.2 to 0.8 and the k-d tree depth was varied from 3 to 5

* System that didn't use the EBBI algorithm produced a 10.3% WER and it used 31.7% of the total CPU time for the Gaussian evaluations

* For a depth of 5, as the threshold increased from 0.2 to 0.8, the Gaussian speedup increased from 39% to 79% and the WER increased from 10.4% to 15.6%

* For a relative threshold of 0.5, as the tree depth increased from 3 to 5, the Gaussian speedup increased from 46% to 58% and the WER increased from 10.9% to 11.2%

AD - 16 Mixture Components (II)



* Best performance — a tree depth of 5 and a threshold of 0.3 produced a 41% speedup with only a 2% relative degradation in WER

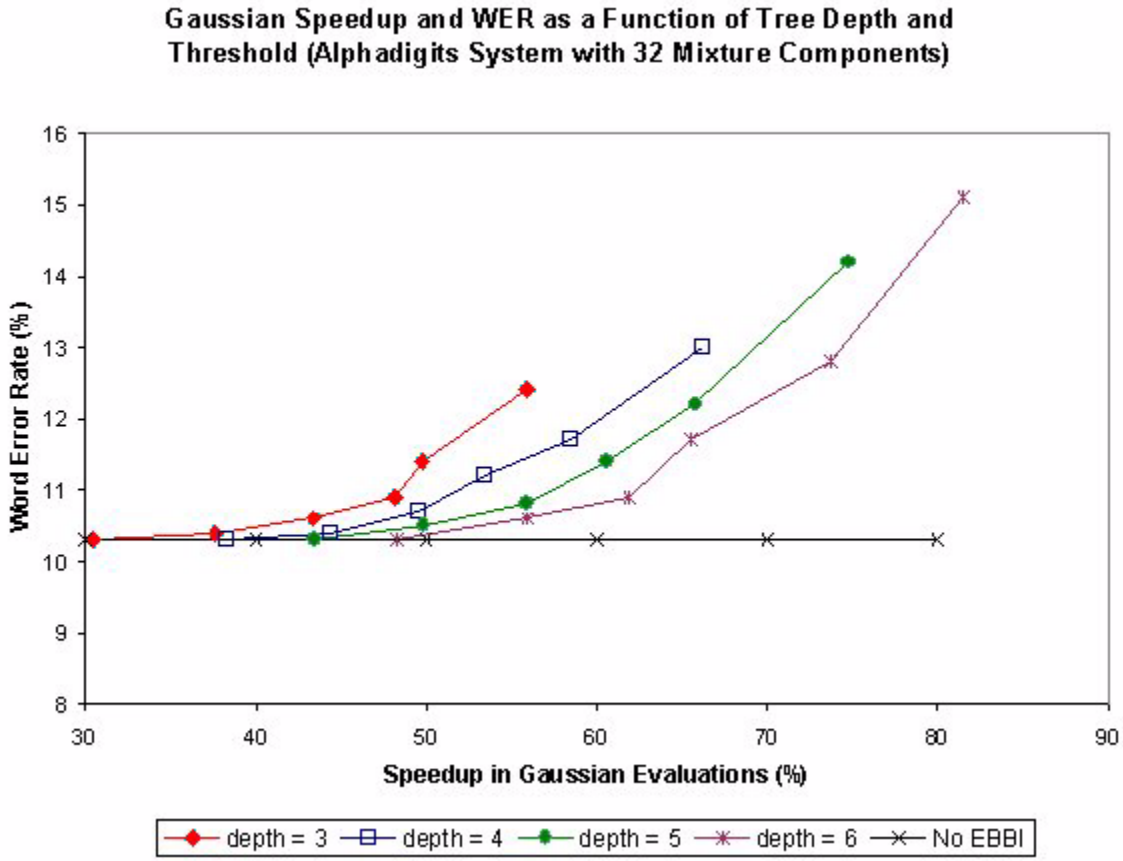
AD - 32 Mixture Components (I)

* The relative threshold was varied from 0.3 to 0.8 and the k-d tree depth was varied from 3 to 6

* System that didn’t use the EBBI algorithm produced a 10.3% WER and it used 43.1% of the total CPU time for the Gaussian evaluations

* The CPU time used by the Gaussian evaluations is 36% higher than the CPU time used by the Gaussian evaluations in the system using 16-mixture components

AD - 32 Mixture Components (II)



* Best performance — a tree depth of 6 and a threshold of 0.3 produced a 48% speedup without degrading the performance of the system

Effect of Mixture Components

* The algorithm becomes more attractive as the number of Gaussians increases

* A 48% speedup is obtained,

  * without any approximation error when 32-mixture components were used

  * with a 4% relative approximation error when 16-mixture components were used

* In general, the algorithm produces the same amount of speedup with a significantly lower increase in WER for the systems with higher mixture components

SWB Database

* HMM system used cross-word context-dependent triphone models
* 39-dimensional MFCC features used
* The relative threshold was varied from 0.3 to 0.8 and the k-d tree depth was varied from 3 to 5
* System that didn’t use the EBBI algorithm produced a 41.1% WER and it used 9.9% of the total CPU time for the Gaussian evaluations
* The EBBI algorithm doubled the speed of the score computation with only a 3.4% relative increase in the WER

Conclusions

* Compared to the BBI algorithm, the EBBI algorithm produced a significantly lower WER (11.4% vs. 10.2%) with only a 1.8% reduction in speedup

* Compared to the GC technique, the EBBI algorithm produced a significantly lower WER (12.2% vs. 10.7%) for a 50% speedup in the likelihood computation

* Algorithm performance for various databases
  * TIDIGITS — a 45% speedup without any degradation in the system performance
  * AD — a 48% speedup without any degradation in the system performance
  * SWB — a 50% speedup with only a 3.4% relative increase in the system WER

Future Work

* Need to study the performance of the algorithm under various possible hyperplanes resulting in a balanced distribution for a coordinate axis
  * the k-d trees obtained by using different hyperplanes may result in a different set of most significant Gaussians
* Include the mixture weights in computing the Gaussian boxes
  * may improve the algorithm performance without an additional computational cost during recognition

References

1. J. Bentley, "Multidimensional Binary Search Trees Used for Associative Searching," *Communications of the ACM*, vol. 18, no. 5, pp. 509-517, September 1975.

2. V. Ramasubramanian and K. Paliwal, "A generalized Optimization of the K-d Tree for Fast Nearest Neighbor Search," *Proceedings of 4th IEEE Region 10 International Conference on TENCON,* pp. 565-568, November 1989.

3. J. Fritsch, and I. Rogina, "The Bucket Box Intersection (BBI) Algorithm for fast approximative evaluation of Diagonal Mixture Gaussians," *Proceedings of International Conference on Acoustics, Speech, and Signal Processing*, vol. 1, pp. 837-840, 1996.

4. J. Fritsch, I. Rogina, T. Sloboda, A. Waibel, "Speeding Up The Score Computation Of HMM Speech Recognizers With The Bucket Voronoi Intersection Algorithm," *Proceeding of the EUROSPEECH,* vol 2, pp. 1091-1094, Madrid, Spain, 1995.