# On the Modeling of TCP Latency and Throughput



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#### **MASTER'S THESIS PRESENTATION**

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# Outline

- Introduction and Motivation
- Background Information on TCP
- Building the Stochastic Models
- Model Validation by Simulation
- Conclusion and Future Work



# What Is The Problem ? - From Practical View

- TCP's performance dominates behavior of Internet traffic inspiring tremendous research on stochastic TCP model
  - 1. *improve TCP performance by understanding the sensitivity of TCP performance to the network conditions*
  - 2. help design of active queue management
  - *3. aid in the design of TCP-friendly transfer multicast protocols*
  - -An accurate model of TCP performance is needed



What Is The Problem ? -From the Model's View

- Most existing models doesn't include the analysis of timeouts effects
- Models including the analysis of time-outs underestimate it
- None of the existing steady state model include the slow start phase
- Not accurate modeling of the delayed acknowledgment's effect in the slow start phase
- New coupled models are needed



What Is This Research All About?

- Develop better and tractable model for slow start
- Develop complete steady state model including the slow start phase
- Develop accurate model for short-lived TCP flows



#### Why Include the Slow Start?

- Slow start phase begins whenever TCP recovers from timeout phase
- Empirical studies observed that slow start phase occurs often for long-lived TCP flows
- Models that ignored slow start overestimate TCP performance
- Including slow start phase into steady state analysis results in accurate performance predictions



Why Need New Models for shortlived TCP connections?

- 85% of TCP traffic are short-lived flows
- Connections ends while in slow-start phase
  - never enter congestion avoidance
  - steady-state model doesn't apply



# **TCP Features**

- •Connection oriented
- Explicit and acknowledged connection establishment
- Reliable stream exchange
- every packet has sequence number
- acknowledging the receipt of the right packet (usually delayed)
- set retransmission timer for every packet sent
- Congestion control



# Slow Start and Congestion Avoidance

If current congestion window (cwnd) is less than slow start threshold (ssthresh)	If (cwnd < ssthresh)
TCP is in slow start phase, and increase the cwnd exponentially	$\operatorname{cwnd} = \operatorname{cwnd} + 1;$
Otherwise in congestion avoidance mode, and cwnd increases linearly	Else cwnd += 1/cwnd;





# **Steady State Model**

- Assumptions

- Based on TCP Reno release from Berkeley
- High link speed
- Fixed packet size

Congestion window alone determines the send rate



# Steady State Model - Assumptions (Continued)

• Modeling dynamics of TCP in terms of "rounds"

— starts when a window of packets is sent and ends when one or more acknowledgments are received

- Delayed acknowledgment algorithm applied
- Packet losses in accordance with bursty loss model

— Packet losses are correlated in each round but independent between rounds





### **Steady State Model**

Let  $M_i$  be the number of packets sent during the total time  $S_i$ :

$$M_{i} = Y_{i}^{SS} + \sum_{j=1}^{n_{i}} Y_{ij} + R_{i}$$
$$S_{i} = Z_{i}^{SS} + \sum_{j=1}^{n_{i}} A_{ij} + Z_{i}^{TO}$$

Assuming (M, S) to be sequences of i.i.d. random variables, the send rate is:

$$B = \frac{E[M]}{E[S]}$$



# **Steady State Model**

Considering  $n_i$  to be i.i.d. random variables and independent of  $Y_{ij}$ , we have:

$$B = \frac{E[Y^{SS}] + E[n]E[Y] + E[R]}{E[Z^{SS}] + E[n]E[A] + E[Z^{TO}]}$$



#### **Slow Start Phase**

- Congestion window growth pattern is:  $cwnd_i = [\frac{cwnd_{i-1}}{2}] + cwnd_{i-1}$
- The total number of packets sent in first n rounds :  $Y_n^{SS} = \sum_{i=1}^n cwnd_i$

Number of packets sent	$E[Y^{SS}] = \frac{E[W^{TD}]g^2}{2} - 2$
Time duration	$E[Z^{SS}] = \log_g(\frac{E[W^{TD}]}{2C_1}) * RTT$



# **Congestion Avoidance Phase**





# Congestion Avoidance Phase (continued)

• Expected congestion window size:

$$E[W^{TD}] = -\frac{2(b-2p)}{3} + \sqrt{\frac{4(bp+2(1-p^2))}{3bp} + (\frac{2b-4p}{3b})^2}$$

• Number of packets sent in the fast retransmit:

$$E[\beta] = (E[W^{TD}] - 1)(1 - p)$$

• Number of rounds in TDP:

$$E[X] = b(\frac{E[W^{TD}]}{2} + 1)$$



# Time-outs Phase (continued)

- Padhye's steady-state model use: 1/E[w] = E[1/w]
- Not so good approximation:

$$E[(\frac{1}{\sqrt{W}})(\sqrt{W})]^{2} \leq E[(\frac{1}{\sqrt{W}})^{2}]E[(\sqrt{W})^{2}]$$
$$\Rightarrow \qquad \frac{1}{E[W]} \leq E[\frac{1}{W}]$$

• Better approximation:

$$E[\frac{1}{W}] \approx \frac{1}{E[W]} (1 + \frac{Var(W)}{E[W]^2})$$



Probability of Packet Loss Resulting in time-out

Q

$$TD = E[Q^{TD}(w)]$$
$$= E[\min(1, \frac{3}{w})]$$
$$= \min(1, 3E[\frac{1}{W^{TD}}])$$
$$\approx \min(1, \frac{3\sqrt{3}}{E[W^{TD}]})$$



# Send Rate and Throughput

• Send Rate:

— is the number of packets sent per seconds

• Throughput:

— is the number of packets received per seconds From Padhye's model:

Number of TDPs in a congestion avoidance phase	$E[n] = \frac{1}{Q^{TD}}$	
Number of packets sent in the time-out phase	$E[R] = \frac{1}{1-p}$	
Time spent in the time-out phase	$E[Z^{TO}] = T_0 \frac{f(p)}{1-p}$	

# **Send Rate**

$$B = \begin{cases} \frac{E[W^{TD}]g^2}{2} - 2 + \frac{1}{Q^{TD}(E[W^{TD}])}(\frac{1-p}{p} + E[W^{TD}]) + \frac{1}{1-p}}{(\log_g(\frac{E[W^{TD}]}{2C_1}) + \frac{1}{Q^{TD}(E[W^{TD}])}(\frac{bE[W^{TD}]}{2} + b + 1))RTT + \frac{f(p)T_0}{1-p}} \\ When \quad E[W^{TD}] < W_m \\ \frac{W_{mg}^2}{2} - 2 + \frac{1}{Q^{TD}(W_m)}(\frac{1-p}{p} + W_m) + \frac{1}{1-p}}{\log_g(\frac{W_m}{2C_1})RTT + \frac{1}{Q^{TD}(W_m)}((\frac{b}{8}W_m + \frac{1-p}{pW_m} + 2) + 1)RTT + \frac{f(p)T_0}{1-p}} \\ When \quad E[W^{TD}] \ge W_m \end{cases}$$



# Throughput

To obtain throughput, changes are needed:

 $E[Y'] = E[\alpha] + E[\beta] - 1$ 

The number of packets that have been sent in a TDP

E[Y]

E[R] The expected number of packets sent in the time out phase The number of packets that have been received in a TDP

E[R'] = 1The expected number of packets received in the time out phase



# Throughput (continued)









# Short-lived TCP Connection Model

- Initial three-way-handshake connection

   modeled by Cardwell's paper
- 2. Initial slow start
  - same model used in steady state model
- 3. First loss
  - same analysis used for time-out phase
- 4. Subsequent losses
  - good approximation: Steady-state model



# Short-lived TCP connection Model

• Time spent in initial slow start part:

 $E[n] = \begin{cases} \left[ \left[ \log_{g} \left( \frac{W_{m}}{C_{1}} \right) \right] + \frac{1}{W_{m}} \left( E[Y_{init}] - g^{2}W_{m} - 2 \right) \right] \\ When \qquad E[W_{init}] > W_{m} \\ \left[ \log_{g} \left( \frac{E[Y_{init}] + 2}{C_{1}} \right) \right] - 2 \\ When \qquad E[W_{init}] \le W_{m} \end{cases}$ 

• Time spent in the first loss part:

 $T_{loss} = (1 - (1 - p)^d)(Q_{init}E[Z^{TO}] + (1 - Q_{init})E[n_t])$ 

• Time spent in the rest part:

$$T_{rest} = \frac{d - E[Y_{init}]}{H}$$
$$= \frac{dp - (1 - (1 - p)d)(1 - p)}{p * H}$$



# Short-lived TCP Connection Latency

$$T_{latency} = E[T_{twhs}] + E[n]RTT + T_{loss} + T_{rest} + T_{delay} - \frac{RTT}{2}$$

- T<sub>delay</sub> : caused by delayed acknowledgment for the first packet which is characterized by mean of 100ms
- Only half of a round is needed to send the last window of packets, so deduct the hlaf round trip time from the total latency.



# Short-lived TCP model —> Steady state model











# Comparison of the Average Error

Loss Rate	$\mathbf{P}=0$	3×10 <sup>-3</sup> ~ 10 <sup>-1</sup>		
File Size	0.5~26KB	2KB	6KB	11KB
[CSA00]	9.40%	4.08%	6.43%	8.38%
Proposed	5.83%	0.59%	7.54%	7.64%



# Conclusions

- Propose new model for the slow start phase
  - Based on discrete equation
  - Using results from Fibonacci sequence
- Develop complete steady state model
  - Integrate slow start phase
  - Accurate time-out analysis
- Develop accurate short-lived TCP model
  - Using same analysis of slow start
  - New estimate time-out analysis



# 1. Future Work

• Considering effect of fast recovery

— will help building a more accurate model

• Analyze effects of different loss models to TCP's performance

— help design different queuing methods

• Find probability distribution of latency

— better than the expected value

