# Linear Dynamic Models for Speech Recognition

Hidden Markov models (HMMs) have been the most popular approach for acoustic modeling in speech recognition along with the diagonal covariance matrix assumption in which correlations between feature vectors for adjacent frames are ignored. Instead, Linear Dynamic Models (LDMs) use a state space-like formulation that explicitly models the evolution of hidden states using an autoregressive process. This smoothed trajectory model allows the system to better track speech dynamics especially for noisy speech signals. In this work, we proposed LDMs as an alternative to hidden Markov models (HMMs) for robust speech recognition in noisy environments. Our evaluation results showed that, for complex recognition task Aurora 4, LDM classifiers achieved a 4.9% relative accuracy increase for the clean evaluation data and a 6.5% relative accuracy increase for the noisy data over a comparable HMM system with 3-state models. Currently we are in the middle of developing a HMM/LDM hybrid decoder architecture to model the frame correlation using LDMs as well as utilizing HMMs techniques for phone segment alignment. Preliminary experiments will be presented on the Alphadigits (AD) and Resource Management (RM) speech corpora.

 **State Space Models**

In the last few decades, a variety of linear Gaussian models have founded many applications in domains such as control, machine learning and financial analysis. As a starting point, it is useful to introduce the idea of a state-space model, in which data is described as a realisation of some unseen process. The following two equations describe a general state space model:



A -dimensional observation is linked to a -dimensional state vector by the first equation, and the state’s evolution is governed by the second equation.

In state space models, the observations are seen as realisations of some unseen, usually lower-dimensional, process. This provides a means of distinguishing the underlying system from the observations which represent it. The state and observation spaces are linked by the transformation . The observation noise characterises the variation due to a range of external sources, for example measurement error or noise. Furthermore it offers a degree of smoothing which is useful when there is a mismatch between training and testing data. Uncertainty in the modelling of the state process is described by the state noise . An important feature of these models is that the observation at time is conditional only on the state at that time. However, the state can take a variety of forms, such as static distributions, long-span auto regressive processes or sets of discrete modes. Figure 1 represents such a model, where motions in the state space give rise to the observed data.

State-space models are useful in many real-life situations where systems contain a different number of degrees of freedom, usually fewer, than the data used to represent them. In these cases, a distinction can be made between the production mechanism at work and the parameterization chosen to represent it. The hidden state variable can have just as many degrees of freedom as are required to model any underlying processes, and then a state-observation mapping shows how these are realised in observation space. This offers a means of making a compact representation of the data. In fact, dimensionality reduction is a common application of this class of models.



*Figure 1: An overview of state space model.*

There are two problems which must usually be solved for practical application of any given state-space model. Firstly, it should be possible to infer information about the internal states of the model for a given set of parameters and sequence of observations. Secondly, the parameters which identify the model must be estimable given suitable training data.

**Linear Dynamic Models**

Linear Dynamic Models (LDMs) are an example of a Markovian state space model, and in some sense can be regarded as analogous to an HMM since LDMs do use hidden state modeling. With LDMs, systems are described as underlying states and observables combined together by a measurement equation. Every observable will have a corresponding hidden internal state. This is illustrated in Figure 2.



*Figure 2: Internal states and observations in a LDM.*

The general LDM process is defined by:

where,

: -dimensional observation feature vectors

: -dimensional internal state vectors

: initial state with mean and covariance matrices

: state evolution matrix

: observation transformation matrix

: uncorrelated white Gaussian noise with mean and covariance matrices

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The LDM assumes that the dynamics underlying the data can be accounted for by the autoregressive state process. This describes how the Gaussian-shaped cloud of probability density representing the state evolves from one time frame to the next. A linear transformation via the matrix and the addition of some Gaussian noise provide this, the dynamic portion of the model. The complexity of the motion that second equation can model is determined by the dimensionality of the state variable, and will be considered below. The observation process shows how a linear transformation with the matrix and the addition of measurement noise relate the state and output distributions.

The system’s hidden states are the deterministic characteristic of an LDM which are also affected by random Gaussian noise [7]. The state and noise variables can be combined into one single Gaussian random variable. Based on Figure 2, the conditional density functions for the states and output can be written as follows:

According to the Markovian assumption, the joint probability density function of the states and observations becomes:

The system’s states are hidden. We need to estimate the hidden state evolution given an N-length observation sequence and the model parameters. This can be accomplished using a Kalman filter combined with a Rauch-Tung-Striebel (RTS) smoother. The Kalman filter provides an estimate of the state distribution at time given all the observations up to and including that time. The RTS smoother gives a corresponding estimate of the underlying state conditions over the entire observation sequence. For the smoothing part, a fixed interval RTS smoother is used to compute the required statistics once all data has been observed.

The RTS smoother adds a backward pass that follows the standard Kalman filter forward recursion [2]. In addition, in both the forward and the backward pass, we need some additional recursions for the computation of the cross-covariance. The RTS equations are:

A synthetic LDM model with two-dimensional states and one-dimensional observations was created to demonstrate the contribution of RTS smoothing. In Figure 3 we show the state predictions of this LDM model using traditional Kalman filter. In Figure 4, the performance of the Kalman filter with RTS smoothing is shown. In both figures, the green lines represent the trajectories of the two-dimensional true state evolution for our synthetic LDM model. The blue points are the scatter plot of the noisy observations of the LDM model.



*Figure 3: A Kalman Filter.*



*Figure 4: A Kalman Filter with an RTS smoother.*

We can see the predicted results roughly simulate the true state evolution. After adding RTS smoothing into the Kalman filtering process, we observe significantly better prediction for the system internal states.

**EM Training and Implementation Issues**

The Expectation-maximization (EM) algorithm [7] is used to find the maximum likelihood estimates of parameters for a specific word or phone, where the model depends on unobserved latent variables. The relevant equations are:

The E step algorithm consists of computing the conditional expectations of the complete-data sufficient statistics for standard ML parameter estimation. Therefore, the E step involves computing the expectations conditioned on observations and model parameters. The RTS smoother described previously can be used to compute the complete-data estimates of the state statistics. EM for LDM then consists of evaluating the ML parameter estimates by replacing and with their expectations.

The EM algorithm converges quickly and is stable for our synthetic LDM model of two-dimensional states and one-dimensional observations. After initilizing this LDM model with an identity state transition matrix and random observation matrix, the first iteration of ML parameter estimation was applied to update the model parameters. Log-likelihood scores of observation vectors were calculated and saved in order to perform further analysis.

EM training was applied for 30 iterations. After the training recursion, intermediate log-likelihood scores of observation vectors for each iteration of LDM were plotted as a funtion of the number of iterations. This plot is is refered as the EM evolution curve. We explored 1-, 4-, 7-, and 10-dimensions for a state in the LDM approach, and applied EM training for each specified dimension. In Figure 4, the EM evolution curve is shown as a function of the state dimension.



*Figure 4: EM evolution vs. state dimension.*

One important practical issue about our EM implementation is that the linear transformation matrix might lead the ML parameter estimation to produce erroneous parameters when . The reason for this is that the LDM state evolution would grow exponentially if the matrix is not a decaying transformation [4]. Such behavior may not be apparent over a small numbers of frames, but it appears quite often when the training dataset gets large, especially in the situation where the state is not reset between models.

In this case, the most common solution is to use Singular Value Decomposition (SVD) to force after each iteration of EM training. SVD provides a pair of orthonormal bases and , and a diagonal matrix of singular values S such that:

Every element of greater than will be replaced by for a small number of (usually ). By adding the SVD component, we attain good model stability for LDM training, as was described in [2].

For a given speech segment, the likelihood that this segment was generated from a specific LDM can be calculated from Kalman filter equations. For a standard Kalman Filter, the state estimation error at time t can be represented as:

After replacing with the observation equation, the error term becomes:

The associated covariance is:

Since errors are assumed uncorrelated and Gaussian, the log-likelihood of an N-length observation sequence given the model parameters can be calculated as

where and are computed as part of the standard Kalman filter recursions. In classification applications, the latter normalization term can be omitted because it is constant [2].

Some researchers report that the state’s contribution to the error covariance is detrimental to classification performance [2]. During EM training, the resulting fluctuations in the likelihoods computed during the segment-initial frames have the most effect on the overall likelihood of shorter phone segments. For shorter speech segments, it is recommended to replace the error covariance calculation:

with

However, our experimental results did not show a performance improvement for shorter speech segments by using this approach. Hence, in the following experiments, the LDM implementations used the traditional error covariance form.

**Pilot Classification Experiments**

Since LDM has proven to be effective on simulated data, a logical next step was to apply it to the classification of phonetic segments in speech. Our first experiment involved evaluating LDM as a classifier on a simple database consisting of a few phones clearly articulated by a small group of speakers. This data was used to gain a better understanding of key algorithm parameters and their impact on convergence. We refer to this data as the sustained phones database.

The sustained phone database is composed of 2 speakers with 3 phones recorded for each speaker. Each speaker produced 0.5 second utterances of the following phonemes: one vowel ‘aa’, one nasal ‘m’ and one fricative ‘sh’ at a sampling rate of 16 kHz. Feature vectors were generated by computing 12 mel-scaled cepstral coefficients and absolute energy. A frame duration of 10 milliseconds and a window duration of 25 milliseconds was used for feature extraction. The training set consisted of 210 examples (70% of the sustained phone database) of 3 phones from two speakers and the test set consisted of 90 examples (30% of the sustained phone database).

After the data recording and feature extraction, we initialized 3 LDMs (phonemes ‘aa’, ‘m’, and “sh”) using the following strategy: state transition matrix as identity matrix; observation matrix as random entries; observation noise covariance as identity matrix; state transition matrix as identity matrix multiplied by a factor 0.1. The EM algorithm was used for training. We observed that EM training converges after approximately 5 iterations. Different dimensionalities of the state-space were examined and we found 13 dimensions were adequate. Increasing the dimensionality of the state-space to 40 did not improve the classification accuracy in this case.

An HMM system with GMMs was built as the benchmark to evaluate LDM as a phoneme classifier. Table 1 summarizes the relative difference in classification accuracy between LDMs and HMMs. We see that the classification accuracy of the LDM system is 98.9%, which outperforms the best HMM baseline classification accuracy of 91.1% (8-mixture). In the next section, we will further assess LDMs on a large vocabulary evaluation corpus Aurora-4.

*Table 5: EM evolution vs. state dimension.*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Model** | **Vowel *aa*** | **Nasal *m*** | **Fricative *sh*** | **Total** |
| HMM (2-mixt) | 66.67 | 70.0 | 96.77 | 77.8 |
| HMM (4-mixt) | 90.0 | 70.0 | 100 | 86.7 |
| HMM (8-mixt) | 100 | 73.33 | 100 | 91.1 |
| LDM | 100 | 96.67 | 100 | 98.9 |

**Aurora Experiments**

Motivated by the encouraging results on the sustained phone classification experiment, we continued to evaluate LDMs on the Aurora-4 large vocabulary evaluation corpus [6]. This corpus is a well established LVCSR benchmark that does not require extensive computational resources. The data was generated from a machine readable corpus of Wall Street Journal news text. The corpus is divided into a training set and an evaluation set. The training set consists of 7,138 utterances from 83 speakers totaling in 14 hours of speech. The evaluation set consists of 330 utterances from 8 speakers. All utterances were generated at 16 kHz.

The HMM system is used to generate alignments at the phone level. Each phone instance is treated as one segment. A total of 40 LDM phone models, one classifier per model, were used to cover the pronunciations. Each classifier was trained using the segmental features derived from 13-dimensional frame-level feature vectors comprised of 12 cepstral coefficients and absolute energy. The full training set has as many as 30k training examples per classifier. Each phone-level classifier is trained as a one-vs-all classifier. The classifiers are used to predict the probability of an acoustic segment.

Table 2 summarizes the results of the Aurora-4 phoneme classification experiments. The baseline system is composed of 3-state HMMs with varying numbers of mixtures. We show results only for 4-mixture GMMs since the performance increase for larger mixtures was only marginal. The HMM system achieves up to 46.9% and 36.8% accuracy for the clean evaluation data and noisy evaluation data respectively. For the noisy evaluation data, six different kinds of noise (Airport, Babble, Car, Restaurant, Street, and Train) were added randomly to better simulate the real world noisy environment.

*Table 2: Classification (% accuracy) results for the Aurora-4 large vocabulary corpus (the relative improvements are shown in parentheses).*

|  |  |  |
| --- | --- | --- |
| **Model** | **Clean Dataset** | **Noisy Dataset** |
| HMM (4-mixt) | 46.9(-) | 36.8(-) |
| LDM | 49.2 (4.9%) | 39.2 (6.5%) |

From Table 2, we can see that the LDM classifiers achieve superior performance to the HMM classifiers with a classification accuracy of 49.2% for the clean evaluation data and 39.2% for the noisy evaluation data. This represents a 4.9% relative and a 6.5% relative increase in performance over a comparable HMM system with 3-state models. We claim that the LDM model generalizes better than HMM across different channel conditions, which makes LDM a noise robust speech recognition technique.

**Ongoing Work with LDM**

We are currently developing a HMM/LDM hybrid decoder architecture to model the frame correlation using LDMs as well as utilizing HMMs techniques for phone segment alignment. Preliminary experiments will be presented on the Alphadigits (AD) and Resource Management (RM) speech corpora. This HMM/LDM hybrid decoder architecture will be a good evaluation of LDMs on continuous speech recognition tasks, and can be compared to other hybrid decoders we have developed that utilize other nonlinear statistical models (e.g., support vector machines and relevance vector machines).

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