SPARSE BAYESIAN METHODS FOR CONTINUOUS SPEECH RECOGNITION

By

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The prominent modeling technique for speech recognition today is the hidden Markov model with Gaussian emission densities. However, they suffer from an inability to learn discriminative information. Artificial neural networks have been proposed as a replacement for the Gaussian emission probabilities under the belief that the ANN models provide better discrimination capabilities. However, the use of ANNs often results in over-parameterized models which are prone to overfitting. Techniques such as cross-validation have been suggested as remedies to the overfitting problem but employing these is wasteful of both resources and computation. Further, cross-validation does not address the issue of model structure and over-parameterization.

Recent work on machine learning has moved toward automatic methods for controlling generalization and parameterization. A model that has gained much popularity recently is the support vector machine (SVM). SVMs use the principle of structural risk minimization to simultaneously control generalization and performance on the training set. A recent dissertation from this university has employed the SVM in a hybrid

framework for speech recognition. While the HMM/SVM hybrid produced a decrease in the error rate, the implementation had some significant shortfalls which we hope to address in this work. First, the SVMs are not probabilistic in nature and, thus, are not able to adequately express the posterior uncertainty in predictions. This is particularly important in speech where there is significant overlap in the feature space. The SVMs also make unnecessarily liberal use of parameters to define the decision region.

In this dissertation, we study a Bayesian model which takes the same form as the SVM model. This model, termed the relevance vector machine (RVM), provides a fully probabilistic alternative to the SVMs. The RVMs have been found to provide generalization performance on par with SVMs while typically using nearly an order of magnitude fewer parameters. Sparseness of the model is automatic using MacKay's automatic relevance determination methods. In this work we propose to develop the first speech recognition system using RVMs. Similar to hybrid HMM/ANN systems, the RVM model will replace the Gaussian density in the HMM models. To accomplish this, we must develop closed-loop training routines which insure convergence and optimality. Computational issues make this an impossibility currently and must be addressed before a scalable system is feasible.

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CHAPTER 1

INTRODUCTION

Spoken communication is the most natural form of information exchange employed by humans. The communication process requires a speaker to encode information into a set of signals (speech production) and a listener to receive those signals (speech perception), recognize (or decode) the components of the signal (often words, as in speech recognition) and infer the implied meaning of the components and take action (speech understanding) [1,2]. The process of human speech recognition often uses a combination of sensory sources including facial gestures, body language, auditory input as well as feedback from the speech understanding facilities to produce an accurate transcription of the speaker's message. However, for our limited purpose of computer speech recognition, we will consider only the problem of converting an acoustic signal (i.e. the speaker's voice) into a stream of words. This problem is akin to communicating over the telephone where the other sensory side-information is not available. Henceforth, we will consider this as the *speech recognition problem* (see [3] for examples of multimodal recognition technology).

general speech problem

typical modeling approach

discriminative information

Proposed approaches

Machine learning moving toward sparse models

HMM/SVM work

Dissertation work.

Dissertation contributions.

CHAPTER 2

STATISTICAL APPROACH TO SPEECH RECOGNITION

In this chapter, we describe the predominant approach to speech recognition. It is a statistical approach and is framed in a maximum likelihood paradigm using hidden Markov models (HMMs) with Gaussian mixture model (GMM) emission distributions to learn the long-range and local phenomena associated with speech patterns. While tremendously successful, a criticism of these systems is that they are not able to adequately model the discriminative information present in the speech signal. Hybrid systems are described which combine the discriminative-modeling power of artificial neural networks and the temporal modeling power of the HMM. The training techniques for these hybrid systems will serve as inspiration for the techniques developed in this dissertation.

2.1. The Speech Recognition Problem

At the heart of computer speech recognition is a pattern recognition problem. It can be stated thusly: given a set of acoustic observations, $\boldsymbol{O} = \boldsymbol{o}_1, \boldsymbol{o}_2, \ldots, \boldsymbol{o}_T$, and a set of models describing acoustic and linguistic patterns, we must determine which patterns were observed and, in doing so, determine which word sequence, $\boldsymbol{W} = w_1, w_2, \ldots, w_M$ was spoken. Four questions quickly arise from this problem statement:

1. How do we obtain the acoustic observations?

2. What model do we use for the acoustic and linguistic patterns?

3. How do we train these models?

4. How do we find the best word sequence when given a new set of observations?

The first of these questions embodies the problem of finding a suitable transformation of the sampled speech signal into a compact feature space which has properties amenable to pattern recognition techniques. The component of a speech system that implements the transformation is the acoustic front-end. Volumes have been written on front-end processing (for example see [4,5]), however, a fairly generic frame-based, cepstral front-end is at the core of most acoustic front-ends for speech recognition and is

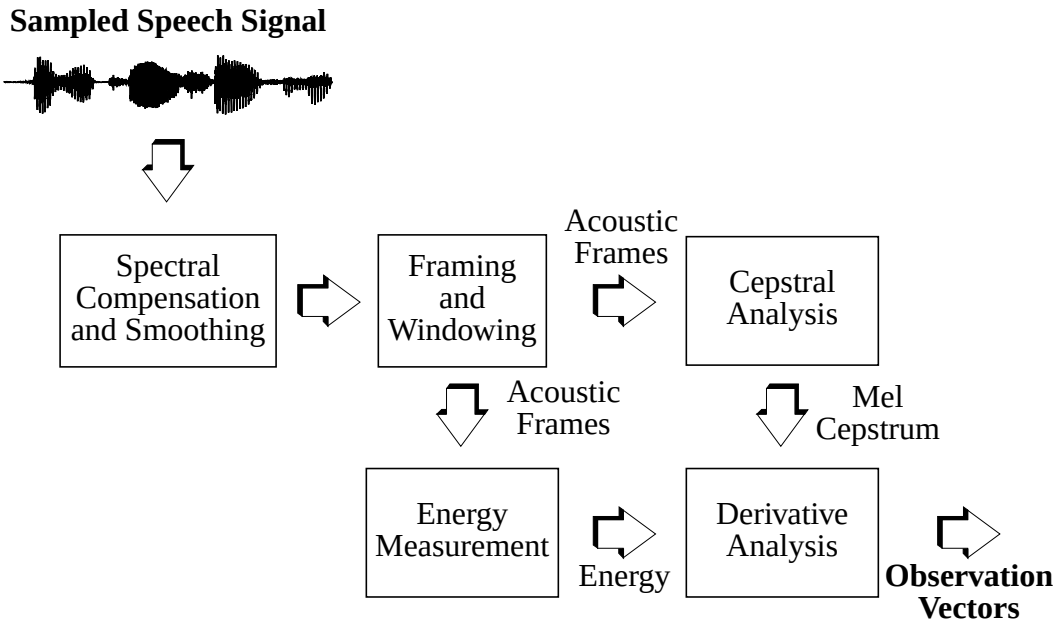

Figure 1. Typical Mel-Cepstral acoustic front-end.

used in this work [6] and is depicted in Figure 1. While this front-end is not the only possibility (see, for example [7]), it has been widely used in speech recognition applications.

At the core of the cepstral front-end is a frame-based analysis which gives a short-time analysis of the sampled speech signal [4]. Under the assumption that the speech signal is stationary over short periods, a frame duration on the order of 10 milliseconds is commonly used. The frame-based approach allows us to analyze the signal in terms of its short-term frequency content. Mel-scale cepstral analysis (MFC) [6] is performed to provide a compact representation of the vocal tract impulse response. The measured cepstral response is correlated with the shape of the vocal tract and position of the articulators at the time at which the frame of speech was uttered. While the frame-based analysis assumes stationarity, it is an unrealistic assumption. Articulators do not instantaneously switch position at frame boundaries, nor are they completely motionless during the frame's duration [8]. To account for some of the transitory behavior, first and second derivative features are typically appended to the feature vector.

With the acoustic observations in place, we can address the second question from above: what model of the acoustic and linguistic patterns do we use? Speech can be loosely seen as a concatenation of units embedded in a hierarchy as shown in Figure 2. For example, we might say that speech is a concatenation of sentences which are, in turn, a concatenation of words which are a concatenation of syllables which, finally, are a concatenation of phones. The phone is often considered to be the smallest, non-divisible unit of sound. In describing the concatenative model, however, we made a false

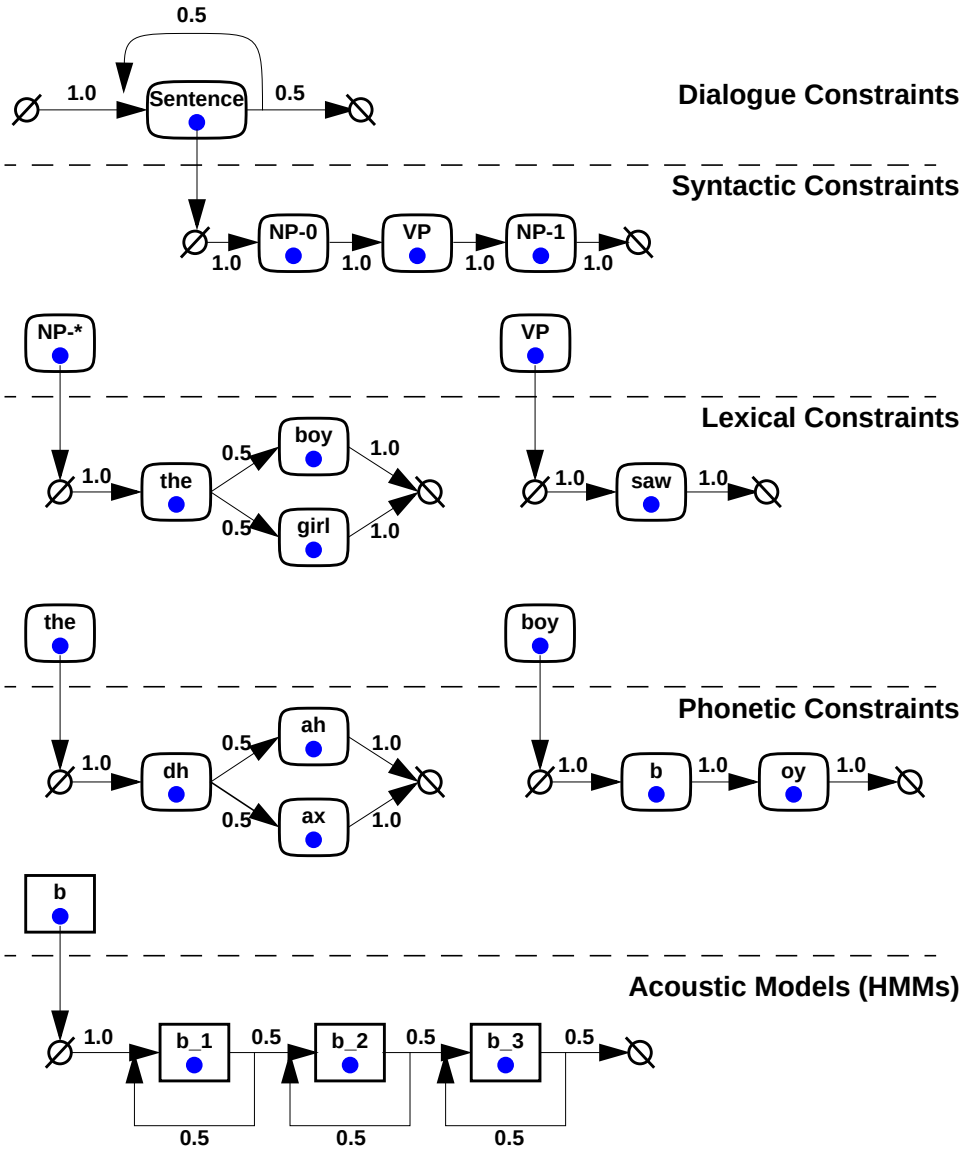

Figure 2. Speech is roughly modeled as a hierarchical constraint system. At each level of the hierarchy, a different knowledge source is applied. The job of a speech recognition system is to combine these knowledge sources in an optimal manner. Often the lowest level in the hierarchy is modeled by hidden Markov models and is responsible for the acoustic match (i.e. modeling the observations sequences generated by the acoustic front-end).

assumption. In conversational speech it is rarely possible to perceptually isolate a single phone. Rather, our perception of a phone is formed from the surrounding phonemic context [9]. For example the ‘a’ sound in the words “am” and “apple” differ — the proximity of the nasal sound, ‘m’ causes the ‘a’ in “am” to be nasalized. This type of effect is particularly prevalent in conversational speech where the speakers are seldom cautious in their articulation [10].

To model these coarticulation effects, we use a context-dependent model in which the model for a base sound is dependent upon the surrounding context. In our example above, the ‘a’ in “am” and the ‘a’ in “apple” would be modeled separately. In most speech applications, a single left context phone and a single right context phone modify the phone in question. This unit is known as a triphone and tends to lead to large increases in performance [11]. Larger contexts have also been applied with some smaller increases in performance [12]. Coarticulation at word boundaries is also a major problem in conversational speech. These effects are modeled by cross-word, context-dependent models.

Speech recognition requires choosing amongst many different possible transcriptions. This requires that we have some principled manner for directly comparing candidate transcriptions so that the “best” one may be chosen. Probabilistic modeling is a natural and very common comparison paradigm and provides our answer to the fourth question above as well: how do we find the best word sequence given a new set of observations. We can reformulate the speech recognition problem as a probabilistic one

where we want to find the word sequence, $\hat{\boldsymbol{W}}$, that is most probable given the acoustic observations, $\boldsymbol{O}$:

$$\hat{\boldsymbol{W}} = \underset{\boldsymbol{W}}{\text{argmax}} \; P(\boldsymbol{W}|\boldsymbol{O}). \tag{1}$$

This *a posteriori* formulation gives us no way to apply information about the *a priori* probability of a word string. Thus, we use Bayes' rule to rewrite (1) as

$$\hat{\boldsymbol{W}} = \underset{\boldsymbol{W}}{\text{argmax}} \; \frac{P(\boldsymbol{O}|\boldsymbol{W})P(\boldsymbol{W})}{P(\boldsymbol{O})} \tag{2}$$

where $P(\boldsymbol{O}|\boldsymbol{W})$ is the probability that the acoustic observations would be seen when a particular word sequence was spoken, $P(\boldsymbol{W})$ is the *a priori* probability of the word string $\boldsymbol{W}$ being spoken, and $P(\boldsymbol{O})$ is the *a priori* probability of the acoustic observation sequence occurring. $P(\boldsymbol{O})$ can be safely eliminated from (2) because the observation sequence, $\boldsymbol{O}$, is constant during the maximization. This yields

$$\hat{\boldsymbol{W}} = \underset{\boldsymbol{W}}{\text{argmax}} \; P(\boldsymbol{O}|\boldsymbol{W})P(\boldsymbol{W}). \tag{3}$$

The terms in (3) are usually modeled separately. $P(\boldsymbol{W})$ is determined by a statistical *language model* which might take the form of a stochastic grammar or an N-gram language model [13,14]. $P(\boldsymbol{O}|\boldsymbol{W})$ is given by an *acoustic model*. This acoustic modeling component of the recognition system is explored in this dissertation. In most state-of-the-art recognition systems, the hidden Markov model (HMM) is used as the statistical acoustic model [15,16,17,18]. The HMM (an example of which is shown in Figure 3) is a doubly-stochastic state machine that can be fully described by the triple

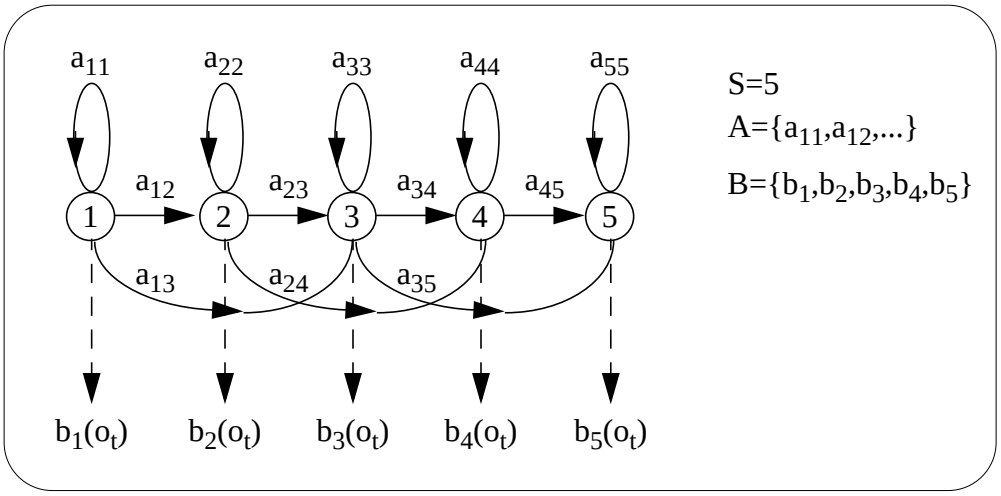

Figure 3. A simple HMM featuring a five state topology with skip transitions. Each state has a stochastic emission distribution.

$\{S, \boldsymbol{A}, \boldsymbol{B}\}$. Here, S is the number of states in the machine, $\boldsymbol{A} = \{a_{ij}\}$ is the state-transition probability set, and $\boldsymbol{B} = \{b_j(\boldsymbol{o}_t)\}$ is the emission probability distribution.

The popularity of HMMs as a model of speech phenomena is owed to the HMMs ability to simultaneously model the temporal progression of speech (speech is usually seen as a "left-to-right" process) and the acoustic variability of the speech observations. The temporal variation is modeled via an underlying Markov process while the emission distribution models the acoustic variability. This acoustic variability may come as a result of differing speakers, channel conditions, stress levels, dialect, accent, etc. in the speech training corpus. The most commonly used emission distribution is the Gaussian mixture model (GMM) described by

$$b_j(\boldsymbol{o}_t) = \sum_{i=1}^{K} C_{ij} N(\boldsymbol{o}_t | \mu_{ij}, \Sigma_{ij}), \quad \sum C_{ij} = 1, \text{ where} \tag{4}$$

$$N(\boldsymbol{o}_t|\mu_{ij},\Sigma_{ij}) = \frac{1}{\sqrt{(2\pi)^n|\Sigma_{ij}|}}\exp\left(-\frac{1}{2}(\boldsymbol{o}_t-\mu_{ij})^{\mathrm{T}}\Sigma_{ij}^{-1}(\boldsymbol{o}_t-\mu_{ij})\right). \quad (5)$$

In (4) and (5), the C_i are the mixture weights and define the contribution of each distribution to the total emission score and n is the dimension of the acoustic observation vector.

Finally, when building the acoustic models with HMMs, one must decide exactly which acoustic unit (e.g. word, syllable or phone) to use. Most state-of-the-art systems, are based on the cross-word context-dependent phones described earlier. In these systems, each context-dependent phone (usually a triphone) is modeled by an HMM. Figure 2 shows how the HMM fits into the hierarchical model described previously.

2.2. Closed-loop, Supervised Parameter Estimation

The answer to the third question above (how do we train these models?) comes from taking an account of the tunable parameters in the hierarchical HMM system described previously. These are the language model probabilities, pronunciation model probabilities and the HMM state transitions ($\{a_{ij}\}$), mixture weights (C_{ij}), means (μ_{ij}) and covariances (Σ_{ij}). We ignore the first two of these in this dissertation and point the reader to [13] regarding training procedures for language models and pronunciation models. Instead, we concentrate on the HMM parameters which are directly related to the acoustic model. Typically, this approach involves finding the HMM parameter set that

maximizes the likelihood of the data given the model, i.e. the maximum likelihood (ML) approach.

As with most machine learning tasks, training acoustic models begins with some labeled training data set. This training set consists of speech data and corresponding word transcriptions (sometimes phonetic transcriptions are available as well). However, in speech, there is a complicating factor: the time alignment of the labels to the speech is usually unknown. For instance, we may be given a five-second segment of speech and told that the transcription is “the boy ate candy”, but we do not know in which time interval each word occurred. Therefore, we can not immediately determine which acoustic observations should be used to train the individual emission probabilities. This is known as the *segmentation problem*.

A simple two-step approach can be taken to overcome the segmentation problem. First, we hypothesize the sequence of HMM states which were most likely to have generated the sequence of acoustic observations given the current parameter set. This is known as a *state-frame alignment.* Second, we update the parameter set according to that state-labeled alignment. This is known as Viterbi training [15] because the first step is a Viterbi alignment of the data to the current model. With this procedure, updating of the HMM/GMM parameters is a straightforward computation of the means and covariances for each GMM given the observations [2].

In the Viterbi training paradigm, a binary decision is made as to whether a state occurred. In other words, the *a posteriori* probability that a particular state generated a particular observation is either 0 or 1. While simple to implement, it is questionable

whether the current model is sufficiently accurate to warrant a hard binary decision or that the iterative procedure will converge. Baum and colleagues [19] addressed these problems by defining a soft-decision training paradigm which is a special case of the expectation-maximization (EM) algorithm [20]. The EM formulation has the desirable property of guaranteed convergence to a local maximum.

Baum [19] defined an EM-type auxiliary function as

$$Q(\lambda, \lambda') = \sum_{\boldsymbol{q}} P(\boldsymbol{O}, \boldsymbol{q}|\lambda')\log P(\boldsymbol{O}, \boldsymbol{q}|\lambda) \tag{6}$$

where λ are the new estimates of the system parameters, λ' are the current system parameters, and $\boldsymbol{q}$ is a given state sequence (i.e. a given state-frame alignment). Maximizing $Q(\lambda, \lambda')$ with respect to λ insures that

$$Q(\lambda, \lambda') \geq Q(\lambda', \lambda') \tag{7}$$

which implies that

$$P(\boldsymbol{O}|\lambda) \geq P(\boldsymbol{O}|\lambda'). \tag{8}$$

Thus, maximizing the auxiliary function monotonically increases the likelihood of the data given the model [2,19,20] until a critical point is reached. Note that the sum over all $\boldsymbol{q}$ in (6) implies a soft decision as to which is the "correct" state-frame alignment. Contrast this to the Viterbi training algorithm where a single alignment was assumed to be the true alignment.

In practice, the Baum-Welch training algorithm is implemented in a forward-backward framework [2,16,17]. We define the forward probability, $\alpha_j(t)$, as the

probability of having observed the partial observation sequence, $\boldsymbol{o}_1, \boldsymbol{o}_2, \ldots, \boldsymbol{o}_t$ and state j at time t:

$$\alpha_j(t) = P(\boldsymbol{o}_1, \boldsymbol{o}_2, \ldots, \boldsymbol{o}_t, q_t = j|\lambda'). \quad (9)$$

We can inductively define $\alpha_j(t)$ as a function of $\alpha_1(t-1), \ldots, \alpha_S(t-1)$. The backward probability, $\beta_j(t)$, is likewise defined as the probability of observing the partial observation sequence, $\boldsymbol{o}_{t+1}, \boldsymbol{o}_{t+2}, \ldots, \boldsymbol{o}_T$, and state j at time t:

$$\beta_j(t) = P(\boldsymbol{o}_{t+1}, \boldsymbol{o}_{t+2}, \ldots, \boldsymbol{o}_T, q_t = j|\lambda'). \quad (10)$$

It can be defined inductively as a function of $\beta_1(t+1), \ldots, \beta_S(t+1)$. These inductive representations provide an extremely efficient method for estimating $\alpha_j(t)$ and $\beta_j(t)$. Note that, in Viterbi training, the $\alpha_j(t)$ and $\beta_j(t)$ were all unity for the states in the assumed alignment and zero for all other state alignments.

The product of $\alpha_j(t)$ and $\beta_j(t)$ gives the probability of any alignment containing state j at time t

$$P(\boldsymbol{O}, q_t = j|\lambda') = \alpha_j(t)\beta_j(t). \quad (11)$$

Likewise, the total probability of observing the sequence, $\boldsymbol{O}$, is just the marginalization across all states at any time

$$P(\boldsymbol{O}|\lambda') = \sum_{j=1}^{S} \alpha_j(t)\beta_j(t). \quad (12)$$

Finally, we can define the probability of any alignment making a transition from state i to state j while observing $\boldsymbol{o}_{t-1}$ in state i and $\boldsymbol{o}_t$ in state j as

$$P(\boldsymbol{O}, q_{t-1}= i, q_t= j|\lambda') = \alpha_i(t-1)a_{ij}b_j(\boldsymbol{o}_t)\beta_j(t). \quad (13)$$

The above three probabilistic equations amount to the expectation step of the EM algorithm. With (11), (12) and (13) in place, we can substitute them into the auxiliary function, (6), and maximize with respect to each model parameter. This process defines the maximization step of the EM algorithm which yields the parameter update equations. These are fully derived in [2,13].

While the combination of HMMs and Gaussian mixture models (HMM/GMM) has been extremely successful, there are some key assumptions made that are not appropriate for modeling of speech symbols.

1. The assumption of conditional independence (i.e. that all probabilities in the system are conditioned only on the current state) is clearly false. The probability of an acoustic observation given a particular state is highly correlated with both past and future observations. Most HMM systems account for this by including derivative features in the observation vector [5], thus breaking the model of conditional independence. Ideally, one would want to condition the distribution itself on the acoustic context, but that is impractical in conventional systems.

2. The HMM/GMM system makes assumptions about the parametric form of the underlying distribution which may lead to a poor match with the true underlying

distribution. To some extent, this can be mitigated by increasing the number of mixtures in the GMM, however.

3. Maximum likelihood approaches do not improve the discriminative abilities of the model. In other words, the ML approach maximizes the probability of the correct model while implicitly ignoring the probability of the incorrect model. Ideally, the training approach should force the model toward in-class training examples while simultaneously driving the model away from out-of-class training examples. Methods such as maximum mutual information [21,22] and minimum classification error [23] have been developed to incorporate discriminative training directly into the standard HMM/GMM framework. However, their success has been limited due primarily to their considerable computational costs [22].

2.3. Connectionist Speech Recognition

The weaknesses of the HMM/GMM system have led researchers to seek models which mitigate some or all of them [24,25,26,27]. Hybrid connectionist systems which merge the power of artificial neural networks (ANNs) and HMMs have received a particularly large amount of attention from the research community in the past decade as an alternative to HMM/GMM systems [24,25,26,28,29,30]. The primary advantages of using the hybrid HMM/ANN systems in speech are:

1. ANNs are trained discriminatively to learn how to not only accept the correct class assignments but to reject the incorrect class assignments.

2. ANN classifiers are able to learn complex probability functions in high-dimensional feature spaces. GMM systems are usually restricted to smaller dimensional vectors (on the order of 30-50) due to amount of training data that would be necessary in estimating the parameters of the GMM distribution. HMM/ANN system designers have put this to good use by using a longer feature vector consisting of a concatenation of the acoustic observations used in the HMM/GMM system; i.e. $\boldsymbol{o}_t^{ANN} = [\boldsymbol{o}_{t-k}, \ldots, \boldsymbol{o}_{t-1}, \boldsymbol{o}_t, \boldsymbol{o}_{t+1}, \ldots, \boldsymbol{o}_{t+k}]$ [24,26]. Note that this also circumvents the independence assumption since consecutive observations for the ANN system are highly correlated.

While some systems have used ANNs to model both the temporal and acoustic properties of speech [31,32], most of the ANN speech systems have used the ANN as a replacement for the GMM probability distribution and have maintained the HMM as a model of the temporal properties. The outputs of a 1-of-N classifier trained under the mean-squared error criteria are known to approximate the posterior class probability, $P(c|\boldsymbol{o})$, where the approximation accuracy is asymptotic in the size of the training set [33]. Recall from the discussion of acoustic modeling earlier that the our goal is to model $\hat{\boldsymbol{W}}$ which maximizes (2). In HMM/GMM systems, we directly build a model of $P(\boldsymbol{O}|\boldsymbol{W})$, but with the ANN systems, we effectively have the posterior phone class probability, $P(C|\boldsymbol{O})$. Thus, the posterior class probabilities need to be converted to likelihoods using Bayes' rule

$$\frac{P(c|\boldsymbol{o})}{P(c)} = \frac{P(\boldsymbol{o}|c)}{P(\boldsymbol{o})}. \quad (14)$$

In practice, the *a priori* class probabilities are estimated from the training data [24,29].

Using (14), the ANN can be used as a direct substitute for the GMM in the HMM framework. Thus, it makes sense that they could/should be trained in the same manner. Initially the hybrid systems were trained using a Viterbi (hard decision) training paradigm as described for HMM/GMM systems above [24,29]. The HMM/ANN system with the current ANN probability estimators was used to create a single alignment of the acoustic observations to the HMM states. The ANN posterior estimators were then trained on each observation that aligned to the HMM state using a typical ANN training algorithm such as back propagation. Parallel training methods were pursued due to the resource-intensive nature of ANN training [34]. Because ANNs are prone to overfitting, a held-out cross-validation set is necessary to test for convergence of the models to a local maxima.

It is well known that, with infinite training data and sufficient model complexity, a neural network trained on binary (0/1) targets will learn the posterior probability distribution perfectly [33]. However, it is less clear how the same ANN will perform when the training data is limited and the model topology is not matched to the true posterior distribution. Yan, et al. [35] claim that, when given unseen data, an ANN trained under such circumstances will produce unreasonable output. An appropriate response would be to make a probability estimate which displays a lack of posterior knowledge about the correct classification (a uniform probability for all classes, for instance). Instead, the ANNs often make extremely confident predictions despite the lack of any prior training

which supports the prediction. To address this issue, researchers have recently begun to explore the use of the Baum-Welch framework as a method for training HMM/ANN hybrids [35,36]. The goal of this method of training the HMM/ANN system is to train the ANN to learn the posterior emission probability distribution from the targets that are readily available from the Baum-Welch procedure:

$$\gamma_j(t) = P(q_t = j|\boldsymbol{O}, \lambda) = \frac{\alpha_j(t)\beta_j(t)}{\sum_{k \in \boldsymbol{S}} \alpha_k(T)}. \quad (15)$$

The ANN is then directly trained on these $\gamma_j(t)$ values.

The HMM/ANN hybrids have shown promise in terms of performance but have not yet found widespread use due to some serious problems. ANNs are prone to overfitting the training data if allowed. To avoid overfitting, a cross-validation set must be used to define a stopping point for the training set. This is wasteful of data and resources — a serious consideration in speech where the amount of labeled training data is very limited. ANNs also typically converge much slower than HMMs. Most importantly, the HMM/ANN hybrid systems have not shown substantial improvements in recognition accuracy over HMM/GMM systems.

2.4. Summary

This chapter has reviewed the most common acoustic modeling framework for speech recognition systems — HMMs with GMM emission probability distributions. The use of ANNs as replacements for the GMM distributions has also been discussed. Of particular importance in this chapter are the training techniques used in the HMM/GMM

systems and the hybrid HMM/ANN systems. The relevance vector machines explored in this dissertation will act in a fashion similar to the ANNs as posterior estimators. Thus, the approaches developed in this dissertation will draw significantly from the HMM/ANN work. However, we will seek methods which are automatically immune to overfitting without the artificial imposition of a cross-validation set as well as methods which can automatically learn the appropriate model structure. The next two chapters define such methods, the support vector machine and relevance vector machine, which both describe principled methods for avoiding overfitting — structural risk minimization for the support vector machine and Bayesian automatic relevance determination for the relevance vector machine.

CHAPTER 3

SUPPORT VECTOR MACHINES FOR SPEECH RECOGNITION

Given a training corpus, $\boldsymbol{O} = \{(\boldsymbol{o}_1, t_1), (\boldsymbol{o}_2, t_2), \ldots\}$ where $\boldsymbol{o}_i$ is the i'th input observation and t_i is the corresponding target (e.g. class assignment or class probability), the goal of a learning machine is to learn the mapping $t = f(\boldsymbol{o})$ under some appropriate optimization scheme. One flexible and popular class of functions are those which are linear combinations of basis functions on the input observations

$$y(\boldsymbol{o};\boldsymbol{w}) = w_o + \sum_{i=1}^{M} w_i \phi_i(\boldsymbol{o}) = \boldsymbol{w}^T \boldsymbol{\phi}(\boldsymbol{o}). \tag{16}$$

A special form of (16) is one in which there is a basis function prescribed for each training vector. These models are generally referred to as *vector machines*. The following chapters discuss and compares two such models: the Support Vector Machine (SVM) [37,38,39,40,41,42,43] and Relevance Vector Machine (RVM) [44,45,46].

3.1. Support Vector Machines

Learning is a process by which a learning machine is optimized under a given set of constraints. We can pose this process as one of optimizing some *risk function*, $R(\alpha)$, where the optimal machine is the one whose free parameters, α, are set such that the risk is minimized. This minimization is written as

$$\hat{\alpha} = \underset{\alpha}{\operatorname{argmin}} \; R(\alpha) = \underset{\alpha}{\operatorname{argmin}} \int Q(\boldsymbol{o}, y, \alpha) dP(\boldsymbol{o}, t) \tag{17}$$

where $Q(\boldsymbol{o}, t, \alpha)$ is a loss function which penalizes the mismatch between both the form and the parameterization of the learning machine and the true function, f; and $P(\boldsymbol{o}, t)$ is the joint distribution of the observations and targets. Finding a minimum for (17) is usually impossible because $P(\boldsymbol{o}, t)$ can not be found *a priori*. Thus, we look for a simplification of (17) that is tractable.

A popular variation of the *actual risk*, $R(\alpha)$, which can be easily evaluated is the measured mean risk, or *empirical risk*, defined as,

$$R_{emp}(\alpha) = \frac{1}{l} \sum_{i = 1 \ldots l} Q(\boldsymbol{o}_i, t_i, \alpha). \tag{18}$$

where l is the number (assumed finite) of training observations. R_{emp} is therefore the loss computed from a fixed training set under the maximum entropy assumption of uniformity for $P(\boldsymbol{o}, t)$. Finding the α which minimizes (18) gives the *empirical risk minimization (ERM)* solution and is one of the most commonly used optimization procedures in machine learning (e.g. mean-square error optimization). However, the issue of the generalization of the learning machine is not specifically addressed when we use ERM. In fact, ERM requires that the training set be representative of the true data distribution to be effective. There could be several settings for the free parameters which give us the same empirical risk. To determine which settings are optimal, we have to know which one would achieve the least actual risk.

Vapnik [37] provides an elegant solution to this problem. Through his analysis of bounds on the actual risk he proved that bounds exist for the actual risk such that,

$$R(\alpha) \leq R_{emp}(\alpha) + f(h) \tag{19}$$

where h is the Vapnik-Chervonenkis (VC) dimension and is a measure of the capacity of a learning machine to learn any training set [37,39] and $f(h)$ is the VC confidence. If $f(h)$ is small (and we have done our job well of fitting the model to the training set), the machine generalizes well because the actual risk is guaranteed to be close to the empirical risk. For binary classifiers where the loss functions are indicator functions, $f(h)$ is defined by

$$\frac{\varepsilon(l)}{2}\left(1 + \sqrt{1 + \frac{4R_{emp}(\alpha_l)}{\varepsilon(l)}}\right) \tag{20}$$

where α_l is the parameter set that defines the learning machine for a particular training set and $\varepsilon(l)$ is the measure of the difference between the actual risk and the empirical risk [49] which we can use to compare system configurations which achieve equivalent empirical risks.

We can write $\varepsilon(l)$ in terms of the VC dimension, h, and the size of the training set, l, as,

$$\varepsilon(l) = 4\frac{h(\log(2l/h + 1)) - \log\eta/4}{l}. \tag{21}$$

From (21), we can see that when l/h is large, ε and $f(h)$ are both small which implies a convergence of the actual risk and the empirical risk [39]. This result matches our

intuition that a less complex machine (i.e. one where the capacity is much smaller than the number of training samples) will generalize better than an overly complex machine given that they achieve the same empirical risk. With this result, we can guarantee both a small empirical risk (training error) and good generalization — an ideal situation for a learning machine. The converse property of (21) is also true — when l/h is small, both ε and $f(h)$ are large and good generalization can not be guaranteed.

The principle of *structural risk minimization* (SRM) [37,49] is formulated to find the minimum point on the curve describing the bound on the expected risk. It provides a principled method to trade-off the accuracy of the trained machine and the complexity of the machine. For a fixed training set size, the VC dimension, h, becomes the controlling parameter in l/h. The joint optimization of R_{EMP} and $f(h)$ is not tractable in practical problems. Thus, the principle of SRM is implemented in one of two distinct ways:

1. Fix the VC confidence to an appropriately low value and optimize the empirical risk.

2. Fix the empirical risk to an appropriately low value and optimize the VC confidence.

The support vector methodology [38,39,41,42,43] implements SRM using the latter approach where the empirical risk is fixed at a minimum (typically zero for separable data sets) and the SVM learning process optimizes for a minimum confidence interval. The SRM principle thus orders the solutions which are optimal in the ERM sense. In the next

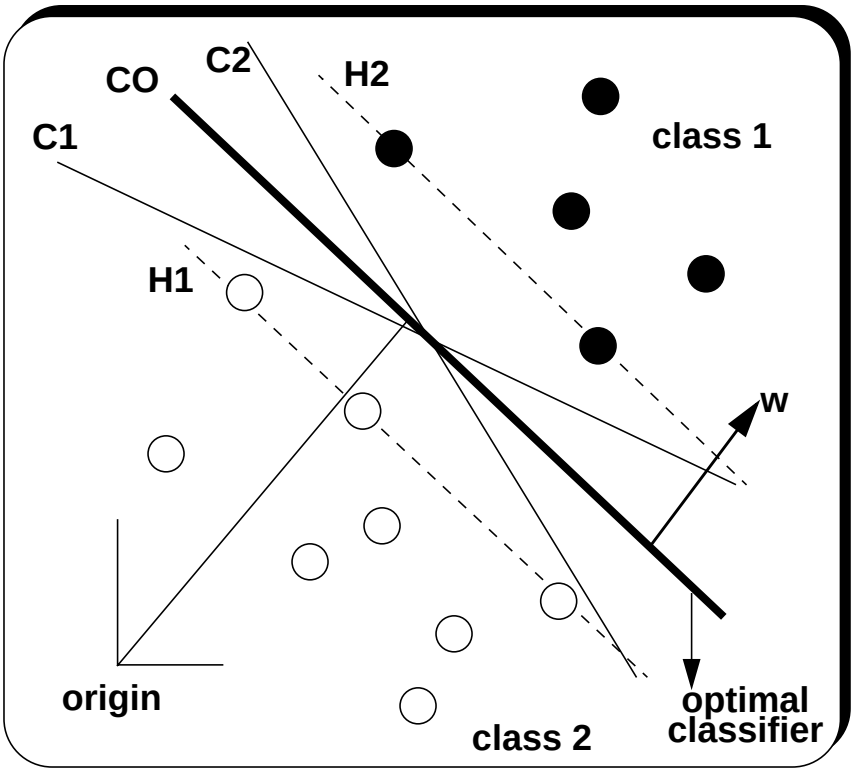

Figure 1. Difference between empirical risk minimization and structural risk minimization for a simple example involving a hyperplane classifier. Each hyperplane ($C0$, $C1$ and $C2$) achieves perfect classification and, hence, zero empirical risk. However, $C0$ is the optimal hyperplane because it maximizes the margin — the distance between the hyperplanes $H1$ and $H2$. Maximizing the margin indirectly results in better generalization.

section, the support vector classifiers will be ordered according to the margin between the class boundaries and the separating hyperplane.

Support Vector Classifiers - Margin Maximization

Figure 1 shows a 2-class classification example where the training samples are linearly separable. H_1 and H_2 define two hyperplanes on which the closest in-class and out-of-class examples lie. The distance separating these hyperplanes is defined as the margin between the two classes. SVMs use the SRM principle to impose an order on the

optimization process by ranking candidate separating hyperplanes based on the margin. For separable data, the optimal hyperplane is the one that maximizes the margin. The existence of a unique hyperplane that maximizes the margin of separation between the classes is guaranteed [37]. The learning procedure is, thus, tasked with finding the location of the optimal hyperplane.

Following [39], let $\boldsymbol{w}$ be a vector that is normal to the separating hyperplane and let $\{\boldsymbol{o}_i, t_i\}, i = 1, \ldots l$ be the training set of length l where $t_i = \pm 1$ indicates class membership (note that this is a binary classification problem with two class indicators, $+1$ and -1). Since $\boldsymbol{w}$ is a normal (not necessarily a unit normal though) to the separating hyperplane, any point, $\boldsymbol{o}$, lying on the separating hyperplane satisfies

$$\boldsymbol{w} \cdot \boldsymbol{o} + b = 0 \tag{22}$$

where $|b| / \|\boldsymbol{w}\|$ is the perpendicular distance of the hyperplane from the origin. We can require that all of the training samples follow the relations

$$\boldsymbol{o}_i \cdot \boldsymbol{w} + b \geq +1 \qquad \text{for } t_i = +1 \tag{23}$$

$$\boldsymbol{o}_i \cdot \boldsymbol{w} + b \leq -1 \qquad \text{for } t_i = -1. \tag{24}$$

These can be combined into a single set of inequalities,

$$t_i(\boldsymbol{o}_i \cdot \boldsymbol{w} + b) - 1 \geq 0 \qquad \forall i. \tag{25}$$

Vectors for which the equality condition in (25) holds are known as *support vectors*.

We can require that all points satisfying the equality condition in (23) lie on the hyperplane $H_1: \boldsymbol{o}_i \cdot \boldsymbol{w} + b = 1$ with normal vector $\boldsymbol{w}$ and distance from the origin of

$|1-b|/\|\boldsymbol{w}\|$. Similarly, all points satisfying the equality condition in (24) lie on $H_2 : \boldsymbol{o}_i \cdot \boldsymbol{w} + b = -1$ and distance from the origin of $|-1-b|/\|\boldsymbol{w}\|$. Relating the distance from the origin of each hyperplane, one can see that the distance between the two hyperplanes (which we defined as the margin earlier) is equal to $2/\|\boldsymbol{w}\|$. Since we are currently only concerned with completely separable data, the margin can be maximized by minimizing $||\boldsymbol{w}||^2$ subject to the constraints of (25). Note that only the support vectors contribute to the SVM solution because it is only those that define the margin. This will become an important property which leads to sparseness in the solution space.

Techniques exist to optimize convex functions with constraints using the theory of Lagrange multipliers [50]. Using these techniques we can pose the functional

$$L_P \equiv \frac{1}{2}\|\boldsymbol{w}\|^2 - \sum_{i=1}^{N} \alpha_i t_i (\boldsymbol{o}_i \cdot \boldsymbol{w} + b) + \sum_{i=1}^{N} \alpha_i \tag{26}$$

which is called the *primal* formulation of the convex optimization problem. Setting the gradient of L_P with respect to $\boldsymbol{w}$ and b to zero gives

$$\boldsymbol{w} = \sum_j \alpha_j t_j \boldsymbol{o}_j, \text{ and} \tag{27}$$

$$\sum_i \alpha_i t_i = 0. \tag{28}$$

Equations (22) and (27) imply that the decision function can be defined as,

$$f(\boldsymbol{o}) = \sum_{i=1}^{N} \alpha_i t_i (\boldsymbol{o}_i \cdot \boldsymbol{o}) + b \tag{29}$$

where the sign of $f(\boldsymbol{o})$ can be used to classify examples as either in-class or out-of-class. This equation defines the SVM classifier. Notice the correspondence between (29) and (16): b corresponds to w_0, α_i to w_i, N to M, and $t_i(\boldsymbol{o}_i \cdot \boldsymbol{o}) = \phi_i(\boldsymbol{o})$. The classifier is completely defined in terms of the training examples and the weights. However only those training examples that lie on the hyperplanes, i.e. the support vectors, define the classifier. In practice, the proportion of the training set that becomes support vectors is small making the classifier sparse. Interestingly, the data set itself defines how complex the classifier needs to be thereby defining the lower limit for the VC confidence, $f(h)$ [39].

Kernel Methods for Nonlinear, Non-separable Decision Problems

The preceding analysis has been only for those problems where the data is linearly separable (i.e. a straight line can be drawn that completely separates the two classes of data). Unfortunately, most real-world data does not conform to this prescription. The data may be nonlinearly separable, or completely inseparable. In either case, we must find a method which optimally bounds the risk while minimizing error on the training set. These problems are attacked with two clever additions to the linear SVM methodology.

In many modeling paradigms, the problem of optimization for non-separable data is solved through the use of soft decision classifiers that place a probability on correctly classifying each training example. However, the SVM is not posed as a probabilistic

problem, so we instead introduce the concept of *slack variables* [38]. The hyperplane constraint equations, (23) and (24), become

$$\boldsymbol{o}_i \cdot \boldsymbol{w} + b \geq +1 - \xi_i \qquad \text{for } t_i = +1, \tag{30}$$

$$\boldsymbol{o}_i \cdot \boldsymbol{w} + b \leq -1 + \xi_i \qquad \text{for } t_i = -1, \text{ and} \tag{31}$$

$$\xi_i \geq 0 \qquad \forall i, \tag{32}$$

where ξ's are the slack variables (one per input observation) that account for training errors since, for an error to occur, ξ_i must exceed unity. Thus, $\sum \xi_i$ gives an upper bound on the number of training errors [38]. A natural way to control the number of training errors is to assign an extra cost for making an error. This is done through the use of a trade-off parameter, C, which is the penalty incurred by the optimizer for accepting a training error. A large value of C will tend to reduce the number of training errors - often at the cost of a more complex model. C is a user-defined parameter that requires a cross-validation procedure to estimate.

Providing for a nonlinear decision region is accomplished using the *kernel* modeling method [51]. Notice that, in the optimization problem formulated in (26), the only place in which the data appears is in the form of dot products, $\boldsymbol{o}_i \cdot \boldsymbol{o}_j$. If we define a transformation of the data to a higher dimensional space by the function $\phi(\boldsymbol{o})$ then we can still construct optimal margin classifiers if we can evaluate the dot product $\phi(\boldsymbol{o}_i) \cdot \phi(\boldsymbol{o}_j)$. It would be highly advantageous if we could define a *kernel* function, K such that

$$K(\boldsymbol{o}_i, \boldsymbol{o}_j) = \phi(\boldsymbol{o}_i) \cdot \phi(\boldsymbol{o}_j). \quad (33)$$

With this function, the dot product in the high-dimensional space could be computed without having to know the explicit form of $\phi(\boldsymbol{o})$. The decision function, (29), then becomes

$$f(\boldsymbol{x}) = \sum_{i=1}^{N} \alpha_i t_i K(\boldsymbol{x}, \boldsymbol{x_i}) + b. \quad (34)$$

Using this kernel method, the SVM is able to transform the training data to a high-dimensional space and construct a linear binary classifier in that space which maximizes a nonlinear margin in the original space. However, only functions which represent a dot product in some space are eligible as kernel functions. Mercer's condition [37] describes the requirements for a function to be a dot product kernel. If a kernel is used which does not satisfy the Mercer conditions, the quadratic optimization is no longer applicable and may lead to a problem whose solution does not converge. Some commonly used kernels include the polynomial and RBF kernels

$$K_{poly}(\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{x} \cdot \boldsymbol{y} + 1)^d \quad (35)$$

$$K_{RBF}(\boldsymbol{x}, \boldsymbol{y}) = \exp\{-\Upsilon|\boldsymbol{x} - \boldsymbol{y}|^2\}. \quad (36)$$

Kernel-based vector machines have had great success on static classification tasks, (those in which no information can be gleaned from the ordering of the exemplars in the input set) for many years (for example [52,53,54,55]) . However, it is only recently that these techniques have been employed on dynamic classification tasks (those in which the

ordering of exemplars is in some way informative) [27,56,57,58]. In this dissertation, we are particularly interested in the application of such machines to the speech recognition problem discussed in Chapter 2. In the remainder of this chapter, we detail the first attempt to apply SVMs to the large vocabulary speech recognition problem using a hybrid HMM/SVM system [27,59,60,61,62].

3.2. Support Vector Methods

Initial attempts to add discriminative information to HMM/GMM speech recognition systems used maximum mutual information (MMI) approaches [21,22] and minimum classification error methods [23]. MMI, in particular, has recently been shown to be quite effective on conversational speech [22]. Later, connectionist systems [e.g. 24,25,26,28,29,30] were employed that used an inherently discriminative ANN acoustic model. While the connectionist systems have been able to match state-of-the-art performance, they did not achieve the great performance gains that were expected on large vocabulary tasks.

New approaches to discriminative modeling for speech recognition have centered around the powerful SVM paradigm described above. The interest in these models for speech is due to two important characteristics of the SVM model. First, SVMs are formulated as optimal generalization machines — overfitting of the data is explicitly avoided in the modeling. Contrast this to neural network approaches where overfitting is typically controlled using a cross-validation process that is wasteful of resources and whose performance is not quantifiable (though see the next chapter for examples of

relevance determination methods by MacKay [47,48] which avoid this problem). This property of SVMs has translated to classification performance that has consistently exceeded neural networks and GMMs [25,53,64]. Second, the SVM (through the use of Mercer kernels) has the ability to build a binary classifier in a high-dimensional space. Unlike other classifiers, neither the dimensionality nor the sparsity of the data in the transform space is a limiting factor for SVMs.

Initial applications of SVMs to speech came in the form of speaker verification systems [65]. Their success was limited, though, due primarily to lack of efficient training methods. Phone classification was the next problem to be tackled using SVMs [59,55]. These systems performed on par with state-of-the-art and their performance was far superior to neural network systems [25] on the same task. With the phone classification problem, the SVM systems were forced to address the first problem with applying SVMs to speech - nonuniform segment lengths. Their solution to this problem was to artificially impose a fixed vector length using a segmental modeling approach that will be described in detail below.

Steps toward using SVMs for word-level continuous speech recognition came in the form of isolated word recognition systems. Bazzi and Katabi [57] built a digit recognition system that employed the same techniques as the phone classification systems. Each digit was modeled with a single one-vs-all classifier. A decimation approach was used to solve the nonuniform segment problem which can be described by the following algorithm:

1. Compute a distance measure, $d_i = f(\boldsymbol{o}_i, \boldsymbol{o}_{i-1})$, for $0 \le i < N$.

2. Find i for which d_i is a minimum. Remove $\boldsymbol{o}_i$ and decrease N by 1.

3. Repeat 1 and 2 until N is the desired size.

Following the decimation stage, a PCA transform was computed to bring the decimated feature vector to its final size. Using a small training set, the SVM system was able to achieve a 5.1% error rate compared to 9.3% error for a GMM classifier. However, state-of-the-art on such tasks is a near-zero error rate.

To move from these simple applications of SVMs as static classifiers to an SVM solution for continuous speech requires addressing two primary issues. First, the dynamic nature of speech must be modeled. SVMs are inherently static classifiers while speech is a dynamically evolving process. The systems described above tried to avoid the problem of dynamics altogether by artificially imposing a fixed vector length. Hybrid connectionist systems address the dynamics of speech by embedding neural networks into an HMM structure [24,29]. The second problem to address is the need to insert SVMs into a probabilistic framework that is used to combine disparate knowledge sources. SVMs are, by definition, binary classifiers capable of giving an in-class/out-of-class judgement. This judgement is rendered by finding the distance from the hyperplane boundary. In general, only the sign of this distance provides useful information, but to apply SVMs in a probabilistic framework one has to map this distance measure to a probability measure (of course one could try to learn the probability function directly using SVM regression but then the power of the discriminative classification is lost).

3.3. Hybrid HMM/SVM System

Research into addressing these remaining issues has proceeded in two directions. First are the systems which use a Fisher kernel capable of handling variable length features [66,67,68] to solve the segmentation problem. While promising, this technique is still in the early stages and has only been applied to relatively simple tasks to date. A more mature method has been defined by Ganapathiraju [27] and colleagues [59,60,61,62,63] which follows a hybrid approach combining techniques from the connectionist systems [24,25,26,29] and segmental modeling systems [69,70]. It is the first to comprehensively address the problems associated with applying SVMs to continuous speech recognition (Chakrabartty, et al. [58] also proposed a hybrid system as well as a circuit design to implement the system in hardware. However, they have only demonstrated their system on a relatively trivial task so it is unclear if their approach holds promise).

Posterior Estimation

The first challenge faced in building the HMM/SVM system is the construction of a probabilistic model from the SVM discriminant function. The approach taken in [27,63] which is drawn from the work of Kwok [71] and Platt [72] is to build a functional mapping from the SVM distance function to a number on the range of [0,1] representing a probability function. If we let $f(\boldsymbol{o})$ be the SVM distance function and t be the class label where $t = \pm 1$, then we can write the posterior probability $P(t = 1 | f)$ as

$$P(t = 1|f) = \frac{P(f|t = 1)P_1}{P(f|t = 1)P_1 + P(f|t = -1)P_{-1}}. \quad (37)$$

It remains, then, to define the form of the likelihood functions, $P(f|t = 1)$ and $P(f|t = -1)$, and the priors on the in-class and out-of-class data, P_1 and P_{-1}.

Taking the maximum entropy approach, the likelihood functions can be defined by Gaussian distributions as

$$P(f|t = 1) = \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{(f-u_1)^2}{2\sigma_1^2}} \text{ and} \quad (38)$$

$$P(f|t = -1) = \frac{1}{\sqrt{2\pi\sigma_{-1}^2}} e^{-\frac{(f-u_{-1})^2}{2\sigma_{-1}^2}}. \quad (39)$$

Normalizing (37) by its numerator and combining exponential terms yields

$$p(t = 1|f) = \frac{1}{1 + \frac{P(f|t = -1)}{P(f|t = 1)}\frac{P_{-1}}{P_1}} = \frac{1}{1 + \frac{P_{-1}}{P_1}\frac{\frac{1}{\sqrt{2\pi\sigma_{-1}^2}} e^{-\frac{(f-u_{-1})^2}{2\sigma_{-1}^2}}}{\frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{(f-u_1)^2}{2\sigma_1^2}}}}, \quad (40)$$

which, after simplification gives the form

$$p(t = 1|f) = \frac{1}{1 + \frac{P_{-1}}{P_1}\frac{\sigma_1}{\sigma_{-1}} e^{-\frac{1}{2}\left[\left(\frac{f-u_1}{\sigma_1}\right)^2 - \left(\frac{f-u_{-1}}{\sigma_{-1}}\right)^2\right]}}. \quad (41)$$

Finally, if we assume that the variances of the discriminant function for in-class and out-of-class data is equal then we can expand the squared terms in the exponent to define the posterior probability in the form of a sigmoid function

$$p(t = 1 \mid f) = \frac{1}{1 + \frac{P_{-1}}{P_1} e^{-\frac{1}{2\sigma^2}((u_1^2 - u_{-1}^2) + 2f(u_{-1} - u_1))}} = \frac{1}{1 + e^{(Af + B)}}. \qquad (42)$$

Here, the parameters A and B are estimated using any suitable nonlinear optimization scheme to optimally map the discriminant function to the probability space. Note that the ratio of the priors has been incorporated into the exponential.

Recall that in the probabilistic formulation of speech presented in Chapter 2 the acoustic model was used to determine the likelihood function; i.e. the probability of the observed data given the assumed model, $P(\boldsymbol{O}|\boldsymbol{M})$. However, from (42), we have derived the posterior estimate of the probability of the model given the data, $P(\boldsymbol{M}|\boldsymbol{O})$. To generate the likelihood function, Bayes' rule needs to be applied. The failure to consider this is a potential weakness in the hybrid HMM/SVM system as it indicates a prior belief that each class is equally likely. Connectionist systems such as those in [24,29] estimate the class priors as part of the training routine. These systems have consistently shown significant degradations in performance when equal priors are applied.

Segmental modeling

A natural way to apply the new SVM acoustic model in an HMM/SVM hybrid system is to perform the classification directly at the frame level — replacing the Gaussian

likelihood score with the SVM posterior described above. In fact, this is exactly the approach used by many hybrid connectionist systems. There are, however, two issues to consider in this regard.

1. **Feasibility for large corpora**: Large vocabulary training sets often contain on the order of 10-100 million frames of speech data. Even with the extremely efficient SVM optimizers available today, it is impractical to train the SVM on this quantity of data. Connectionist systems face a similar problem in the iterative methods used for training [29]. However, parallel processing techniques [34] have been developed that allow them to use large data sets efficiently.

2. **Modeling long-term temporal structure**: Using frame-level data provides a very localized view of the speech signal. It removes the potential for modeling long-range dependencies in data such as cross-frame spectral correlations and for modeling long range "features" of the data such as phone duration [73,74,75]. A few approaches have been tried to alleviate this problem. HMM systems often include derivative terms in the feature stream to account for changes in the feature across frame boundaries [4]. Connectionist systems often concatenate a window of frames around the frame of interest to create a large feature vector [26]. The neural network is then allowed to learn the long-range correlations in the data. HMM/GMM systems could not use such an approach because the number of parameters grows linearly with the size of the feature vector. However, many systems are now using feature reduction techniques such as LDA and PCA to provide the HMM/

GMM systems with a reduced-sized feature vector that still captures the most important long-range correlations [76].

To address both of these issues, the HMM/SVM system uses a segment-based approach akin to those in [69,70]. By modeling at a phone-segment level (i.e. each observation represents a sequence of frames that constitute a single spoken phone), the HMM/SVM system is able to greatly reduce the number of training vectors (by as much as 2-3 orders of magnitude) and is able to simultaneously model both the spectral and temporal structure of speech. With this approach, however, there remains the question of where to get the phone segments in the first place. The HMM/SVM system uses an HMM/GMM system to produce the segmentation information and then post-processes the data under the assumption that the segmentation is correct. Recent linguistic analysis seems to indicate that this is not a good assumption [77] for conversational speech.

Phone segments can have widely varying lengths (e.g. vowels tend to be longer and consonants tend to be shorter). However, with the conventional SVM model (in contrast to those which use Fisher kernels [66,67,68]) we require a fixed observation vector length. One way to mitigate this problem which follows the motivation of 3-state HMM phone models is to divide each segment into a fixed number of distinct subsections [78,79,80]. The frames in each subsection are then averaged and the averages are concatenated to yield a single fixed-length vector. This process is illustrated in Figure 2. While the percentage of the segment that is allocated to each subsection can be

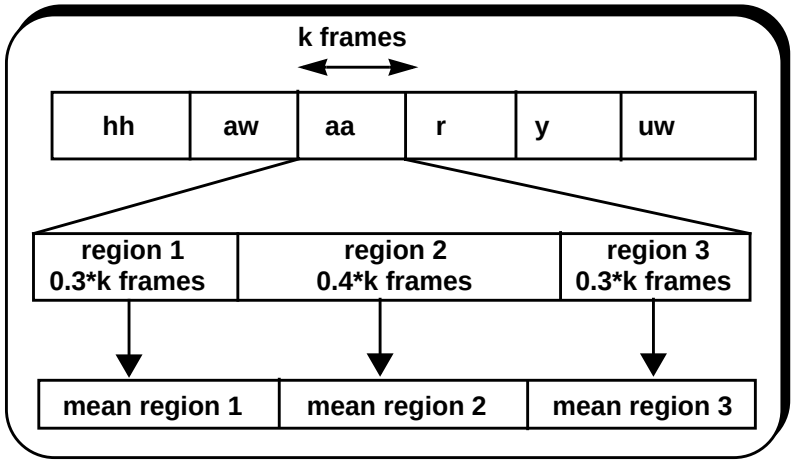

Figure 2. Composition of the segment level feature vector assuming a 3-4-3 proportion for the three sections.

manipulated, the performance of the HMM/SVM system is insensitive to changes in the proportions [27,63].

System architecture

The hybrid HMM/SVM system is built using the rescoring paradigm shown in Figure 3. The HMM/GMM system generates a pruned hypothesis space as well as a segmentation (or set of segmentations). The SVM is used to rescore the hypothesis space given the segmentation(s). In [27,63] N-best lists are used to represent the pruned hypothesis space. These give a set of N unique hypotheses which are most highly predicted by the HMM/GMM system.

For experimental purposes, the segment information was generated in two ways. First, a single segmentation (1-best segmentation) was used to rescore all of the N-best hypotheses. This segmentation was derived from a forced-alignment of a word sequence to the speech data using the HMM/GMM system. For baseline testing, the word sequence

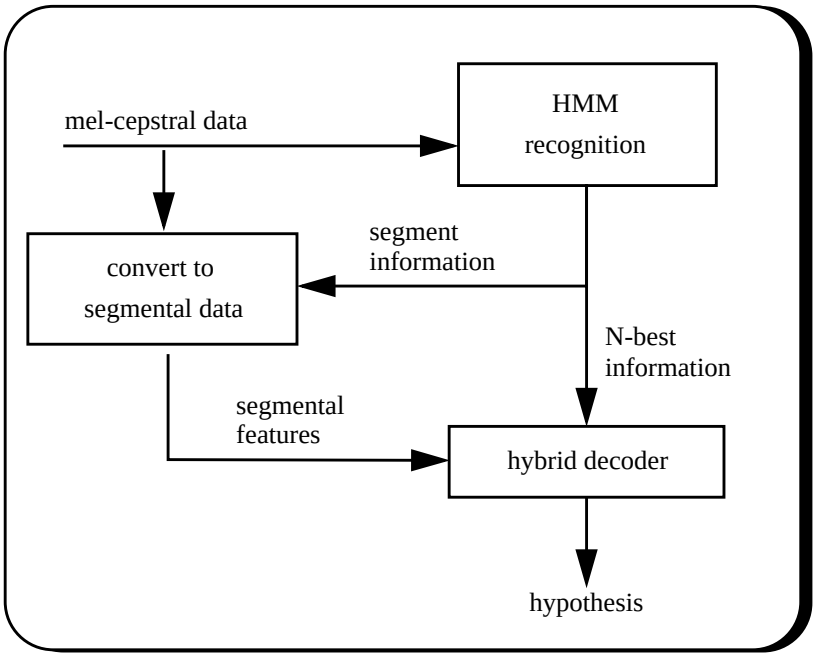

Figure 3. Flow graph for hybrid HMM/SVM system [27].

is the 1-best hypothesis (hypothesis segmentation). This gives the best-guess segmentation of the HMM/GMM decoder. Note, however, that it may not be possible to align some of the N-best hypotheses to the 1-best segmentation, thus the 1-best segmentation acts to artificially constrain the search space for the SVM. For analysis, an oracle experiment can also be run which uses the reference transcription to find a single segmentation. An alternative segmentation method generates a separate segmentation for each entry in the N-best list (N-best segmentation) and rescores each one in turn. While more computationally expensive, this method provides a better comparison with an HMM/ GMM system where the decoder is allowed to choose any segmentation for the hypotheses.

No.	Information Source		HMM		Hybrid	
	Transcription	Segmentation	AD	SWB	AD	SWB
1	N-best	Hypothesis	11.9	41.6	11.0	40.6
2	N-best	N-best	12.0	42.3	11.8	42.1
3	N-best + Ref.	Reference	—	—	3.3	5.8
4	N-best + Ref.	N-best + Ref.	11.9	38.6	9.1	38.1

Table 1. Summary of recognition experiments for hybrid HMM/SVM system [27]. The experiments are differentiated by the corpus (Alphadigits or Switchboard), segmentation type (single segmentation or n-best segmentation) and n-best rescoring type (n-best or oracle n-best + ref). All results are word error rates.

Experimental analysis

The HMM/SVM system was run on two different telephone-bandwidth tasks: the OGI Alphadigits [81] and the SWITCHBOARD (SWB) corpus [82]. The Alphadigits task is a small vocabulary (~40 words), open grammar (any word sequence is possible) task while the SWB task is a large vocabulary (modern lexicons contain as many as 100,000 words) open grammar task. The results of these experiments are shown in Table 1 [27].

The most interesting thing to note about these results is the surprisingly large gains made by the oracle system (experiment 4) for the Alphadigit system. A nearly 30% reduction in WER is achieved by the HMM/SVM system over the HMM/GMM system. This shows the potential power of the SVM classifier when it is presented with adequately rich information from the HMM system. Of course, reducing the n-best list error rate to 0% is usually not possible so we need to look for other ways to give the classifier a wider

variety of hypotheses to choose from — e.g. integrating the SVM directly into the search. Another key point to note is the performance of the oracle system (experiment 3) using the reference segmentation. With this system, a 80% reduction in WER was achieved. While there is no fair comparison to an oracle HMM system given, this performance seems to establish that a good segmentation is the most important issue in applying SVMs in the hybrid framework. Making better use of the HMM framework for temporal modeling and to drive the SVM models is necessary to approach these levels of performance.

A follow-up experiment run as part of this dissertation also showed that the sigmoid posterior estimate applied by the hybrid HMM/SVM system does not significantly contribute to the performance of the hybrid system. In the experiment, the posterior estimate was replaced with a simple thresholding rule that mapped the SVM distance to the range of [0,1]. If the distance was greater than 0 (indicating a sample classified on the in-class side of the decision surface) then a probability of 1.0 was emitted. Otherwise a probability of 0.0 was emitted. In other words, the threshold probability mapping assumes perfect confidence in the classification provided by the SVM. With this modification, the total word error rate on the Alphadigits data was reduced by only 1.8% relative to the HMM/SVM system. If the sigmoid were an accurate model of the posterior, we would expect a more pronounced difference.

3.4. Summary

In this chapter, we have seen how the SVMs use a structural risk minimization argument to define an *optimal* decision surface which automatically rejects overfitting. In

this way, the SVM combines the problems of prediction and decision-making. The theory of Mercer kernels are incorporated into the SVM framework to provide for extremely flexible and highly nonlinear decision surfaces. Further, the chapter has discussed the use of SVMs as classifiers for speech data. The first credible attempt at this is in the form of a hybrid HMM/SVM system. This system uses segmental modeling and posterior estimation techniques to address the issues related to interfacing SVMs to the HMM framework.

In the next chapter we will discuss the relevance vector machine (RVM) which is the object of this dissertation. RVMs use a mathematical structure that is similar to the SVM, but the RVM follows a more conventional motivation. RVMs seek to determine the posterior likelihood of a class assignment given the data, thus allowing for an external decision process. In this way, the RVM can take into account asymmetric misclassification costs, and varying class prior probabilities. Overfitting is avoided through the application of MacKay's ARD principle [47,48]. While the generalization capability of the RVM is comparable to that of the SVM, the RVM offers a few very important advantages which will be explored in this dissertation.

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